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LIGHT WAVES AND THEIR USES

MICHELSON

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THESE VOLUMES ARE DEDICATED

TO THE MEN AND WOMEN
OF OUR TIME AND COUNTRY WHO BY WISE AND GENEROUS GIVING
HAVE ENCOURAGED THE SEARCH AFTER TRUTH
IN ALL DEPARTMENTS OF KNOWLEDGE

LIGHT WAVES AND THEIR USES

LIGHT WAVES AND THEIR USES

BY

A. A. MICHELSON

OF THE DEPARTMENT OF PHYSICS

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THE DECENNIAL PUBLICATIONS
SECOND SERIES VOLUME III

CHICAGO
THE UNIVERSITY OF CHICAGO PRESS
1903

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PREFACE

THIS series of eight lectures on "Light Waves and Their Uses" was delivered in the spring of 1899 at the Lowell Institute. In the preparation of the experiments and the lantern projections I was ably assisted by Mr. C. R. Mann, to whom I am further indebted for editing this volume.

I have endeavored, possibly at the risk of inelegance of diction, to present the lectures as nearly as possible in the words in which they were originally given, trusting that thereby some of the interest of the spoken addresses might be retained.

While it is hoped that the work will be intelligible to the general reader, it is also possible that some of the ideas may be of interest to physicists and astronomers who may not have had occasion to read the somewhat scattered published papers.

A. A. MICHELSON.

RYERSON PHYSICAL LABORATORY
The University of Chicago
October, 1902

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LECTURE I

WAVE MOTION AND INTERFERENCE

SCIENCE, when it has to communicate the results of its labor, is under the disadvantage that its language is but little understood. Hence it is that circumlocution is inevitable and repetitions are difficult to avoid. Scientific men are necessarily educated to economize expression so as to condense whole sentences into a single word and a whole chapter into a single sentence. These words and sentences come to be so familiar to the investigator as expressions of summarized work — it may be of years — that only by considerable effort can he remember that to others his ideas need constant explanation and elucidation which lead to inartistic and wearying repetition. To few is it given to combine the talent of investigation with the happy faculty of making the subject of their work interesting to others. I do not claim to be one of these fortunate few; and if I am not as successful as I could wish in this respect, I can only beg your indulgence for myself, but not for the subject I have chosen. This, to my mind, is one of the most fascinating, not only of the departments of science, but of human knowledge. If a poet could at the same time be a physicist, he might convey to others the pleasure, the satisfaction, almost the reverence, which the subject inspires. The æsthetic side of the subject is, I confess, by no means the least attractive to me. Especially is its fascination felt in the branch which deals with light, and I hope the day may be near when a Ruskin will be found equal to the description of the beauties of coloring, the exquisite gradations of light and shade, and the intricate wonders of symmetrical

forms and combinations of forms which are encountered at every turn.

Indeed, so strongly do these color phenomena appeal to me that I venture to predict that in the not very distant future there may be a color art analogous to the art of sound — a *color music*, in which the performer, seated before a literally chromatic scale, can play the colors of the spectrum in any succession or combination, flashing on a screen all possible gradations of color, simultaneously or in any desired succession, producing at will the most delicate and subtle modulations of light and color, or the most gorgeous and startling contrasts and color chords! It seems to me that we have here at least as great a possibility of rendering all the fancies, moods, and emotions of the human mind as in the older art.

These beauties of form and color, so constantly recurring in the varied phenomena of refraction, diffraction, and interference, are, however, only incidentals; and, though a never-failing source of æsthetic delight, must be resolutely ignored if we would perceive the still higher beauties which appeal to the mind, not directly through the senses, but through the reasoning faculty; for what can surpass in beauty the wonderful adaptation of Nature's means to her ends, and the never-failing rule of law and order which governs even the most apparently irregular and complicated of her manifestations? These laws it is the object of the scientific investigator to discover and apply. In such successful investigation consists at once his keenest delight as well as his highest reward.

It is my purpose to bring before you in the following lectures an outline of a number of investigations which are based on the use of light waves. I trust I may be pardoned for citing, as illustrations of these uses, examples which are taken almost entirely from my own work. I do this because

I believe that I shall be much more likely to interest you by telling what I know, than by repeating what someone else knows.

In order to discuss intelligently these applications of light waves, it will be necessary to recall some fundamental facts about light and especially about wave motion. These facts, though doubtless familiar to most of us here, need emphasis and illustration in order that we may avoid, as far as possible, the tedious repetition against which we were warned.

Doubtless there are but few who have not watched with interest the circular waves produced by a stone cast into a still pond of water, the ever-widening circles, going farther and farther from the center of disturbance, until they are lost in the distance or break on the shore. Even if we had no knowledge of the original disturbance, its character, in a general way, might be correctly inferred from the waves. For instance, the direction and distance of the source can be determined with considerable accuracy by drawing two lines perpendicular to the front of the wave; the source would lie at their intersection. The size of the waves will give information concerning the size of the object thrown. If the waves continue to beat regularly on the shore, the disturbance is continuous and regular; and, if regular, the frequency (*i. e.*, the number of waves per second) determines whether the disturbance is due to the splash of oars, to the paddles of a steamer, or to the wings of an insect struggling to escape.

In a precisely similar manner, though usually without conscious reasoning about the matter on our part, the sound waves which reach the ear give information regarding the source of the sound. Such information may be classified as follows:

1. Direction (not precise).
2. Magnitude (loudness).

3. Frequency (pitch).
4. Form (character).

Light gives precisely the same kinds of information, and hence it is only natural to infer that light also is a wave motion. We know, in fact, that it is so; but before giving the evidence to prove it, it will be well to make a little preliminary study of the chief characteristics of wave motion.

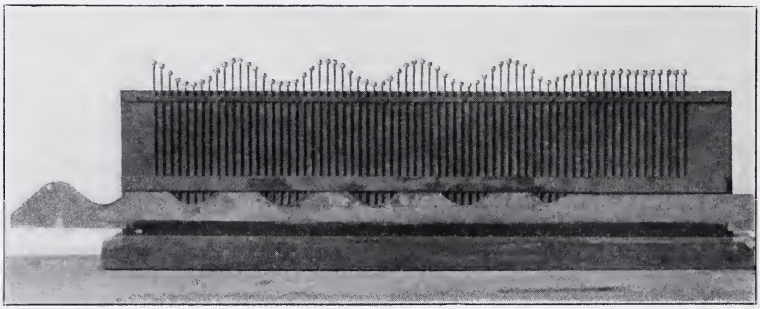


FIG. 1

One of the difficulties encountered in studying wave motion is the rapidity of the propagation of the waves. A fairly moderate speed is attained by the waves propagated along a spiral spring. If one end of such a spring be fastened to a wooden box on the wall of the lecture-room, while the other end is held in the hand, we can see that any motion communicated by the hand is successively transmitted to the different parts of the spring until it reaches the wall. Here it is reflected back toward the hand, but with diminished amplitude. We can also see that any kind of transverse motion, *i. e.*, motion at right angles to the length of the spring, whether regular or irregular, gives rise to a corresponding wave form which travels along the spring with a velocity that is the same in every case.

If the spring be very suddenly stretched or relaxed, a

wave of longitudinal vibrations passes along it, announcing its arrival at the other end by a sound at the box; the time occupied in the passage being perceptibly less than that required for the transverse wave.[1]

The velocity of the wave is in both cases too great to admit of convenient investigation. In order to familiarize the student with wave motion, a number of mechanical devices have been constructed, such as that shown in Fig. 1. Such mechanical models imitate wave motions rather than produce them. They are purely kinematic illustrations, and not true wave motions; for in the latter the propagation is determined by the forces and inertias which exist within the system of particles through which the wave is moving.

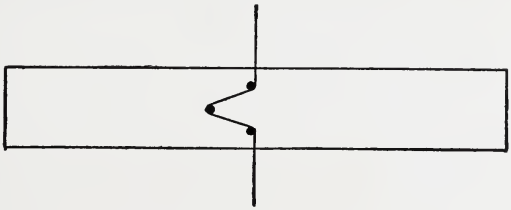


FIG. 2

The wave model of Lord Kelvin is free from this objection. It consists of a vertical steel wire on which blocks of wood are fastened at regular intervals. It is very essential that these blocks should not slip on the wire, and this end is best accomplished by bending the wire, in the middle of each block, around three small nails, as shown in Fig. 2. For the sake of symmetry two such pieces may be fastened together, with the wire passing between them. Attention may be fixed upon the motion of the ends of the blocks, by driving into them large, gilt, upholstering tacks—a device which adds considerably to the attractiveness of the experiment. The complete apparatus is shown in Fig. 3.

On giving the lowest element a twist, the torsion produced in the wire will communicate the twist to the next element, etc. The twist thus travels along the entire row,

[1] I am indebted to Professor Cross for this illustration.

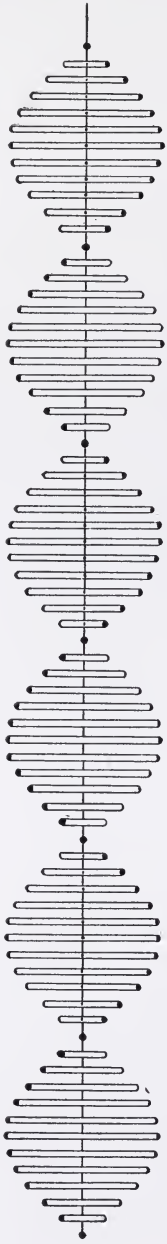


FIG. 3

moving more slowly the smaller the wire and the heavier the blocks, so that, by varying these two factors, any desired speed may be obtained.

The wave form which is propagated in any of the various possible cases is, in general, very complicated. It can be shown, however, that it is always possible to express such forms, however complex, by a series of simple sine curves such as that represented in Fig. 4. The study of wave motion may be much simplified by this device. Accordingly, in all that follows, except where the contrary is expressly stated, it will be assumed that we are dealing with waves of this simple type.

There are certain characteristics of wave motion of which we shall have to speak frequently in what follows, and which therefore need definition. In the first place, the shape of the wave illustrated in Fig. 4 is important. It is the curve which would be drawn by a pendulum, carrying a marker, upon a piece of smoked glass moving uniformly at right angles to the motion of the pendulum. Since the pendulum moves in what is called *simple harmonic motion*, the curve is called a simple harmonic curve, or a sine curve. The *amplitude* of the wave is the maximum distance of a crest or a trough from the position of rest, *i. e.*, from the straight line drawn through the middle of the curve. The *period* of the vibration is the time it takes one particle to execute one complete vibration; *i. e.*, to revert to the pendulum, it is the time it takes the pendulum to execute

one complete swing.[1] The *phase* of any particle along the curve is the portion of a complete vibration which the particle has executed. The *wave length* is the distance between two particles in the same phase. Thus it is the distance

FIG. 4

between two consecutive crests or between two consecutive troughs. When all the particles vibrate in one plane, *e. g.*, the plane of the drawing, the wave is said to be *polarized* in a plane. The *velocity* of propagation of the wave is the distance traveled by any given crest in one second.

As has just been stated, the type of wave motion illustrated in Fig. 4 may be approximately realized by imparting the motion of a pendulum or a tuning-fork to one end of a very long cord. It can be shown that after a time every particle of the cord will vibrate with precisely the

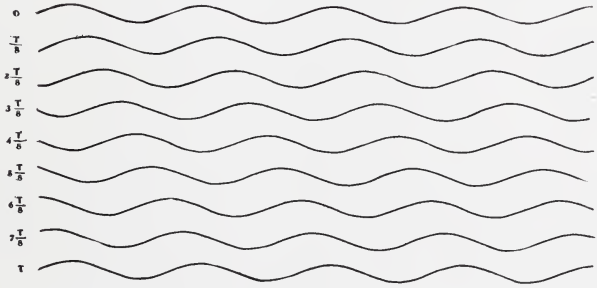


FIG. 5

same motion as that of the pendulum or tuning-fork from which the disturbance starts. Any particular *phase* of the motion occurs a little later in every succeeding particle; and it is this transmission of a given phase along the cord which constitutes the wave motion.

[1] In some works the half of this is taken, *i. e.*, the time it takes a pendulum to move from the extreme left to the extreme right.

Very elementary considerations show that the *length* (l) of the wave is connected with the *period* (p) of vibration of the particles (the time of one complete cycle) and the *velocity* (v) of transmission by the simple relation $l = pv$.

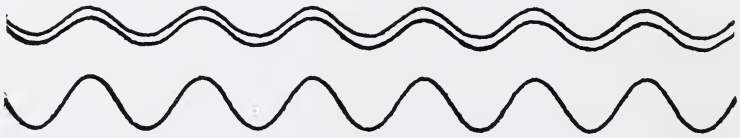


FIG. 6

In fact, if we could take instantaneous photographs of such a train of waves at equal intervals of time, say one-eighth of the period, they would appear as in Fig. 5. It will readily be seen that in the eight-eighths of a period the wave has advanced through just one wave length, while any particle has gone once through all its phases.

Let us next consider the superposition of two similar trains of waves of equal period and amplitude. If the phases of the two wave trains coincide, the resulting wave train will have twice the amplitude of the components, as shown in Fig. 6. If, on the other hand, the phase of one train is half a period ahead of that of the other, as in Fig. 7, the resulting ampli-

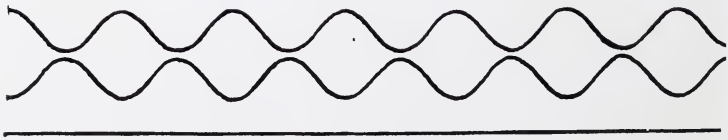


FIG. 7

tude is zero; that is, the two motions exactly neutralize each other. In the case of sound waves, the first case corresponds to fourfold intensity, the second to absolute silence.

The principle of which these two cases are illustrations is miscalled *interference;* in reality the result is that each wave motion occurs exactly as if the other were not there to inter-

fere. The name has, however, the sanction of long usage, and will therefore be retained. The principle of interference is of

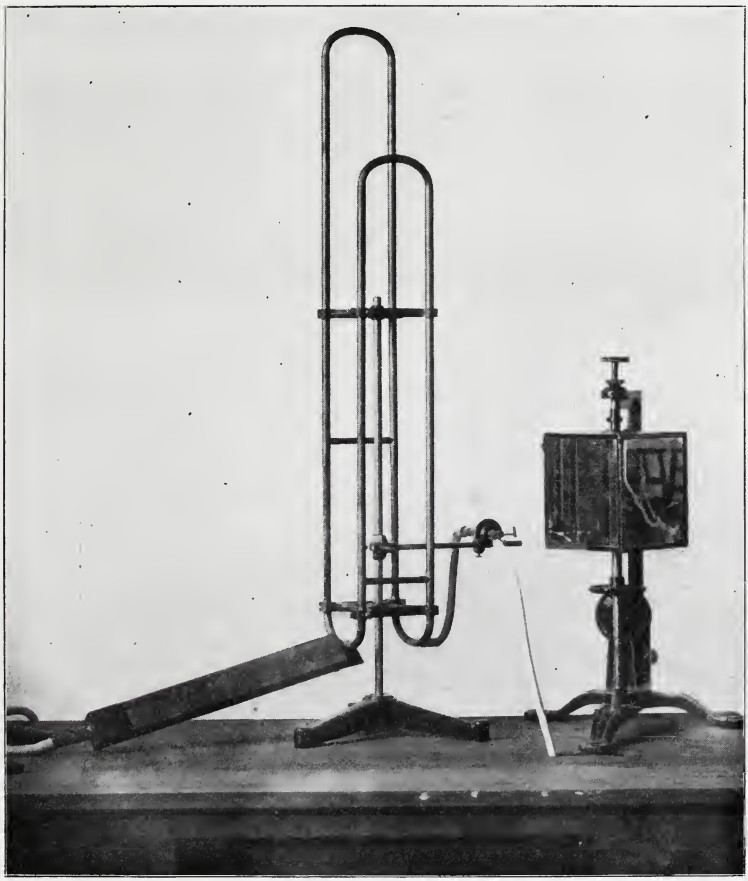


FIG. 8

such fundamental importance that it will be worth while to impress it upon the mind by a few experimental illustrations.

Fig. 8 represents an apparatus devised by Professor Quincke for illustrating interference of sound. An organ

pipe is sounded near the base of the instrument. Thence the sound waves are conducted through the two vertical tubes, one of which is capable of being lengthened, like a trombone. They then reunite and are conducted by a

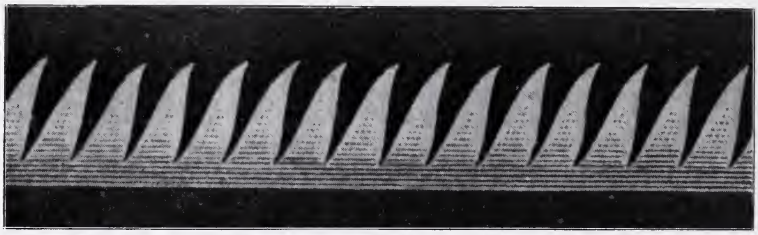


FIG. 9

single tube to a "manometric capsule," which impresses the resulting vibrations on a gas jet, the trembling of the jet being rendered visible in a revolving mirror.

When the two branch tubes are of equal length, the waves reach the flame in the same phase, causing it to

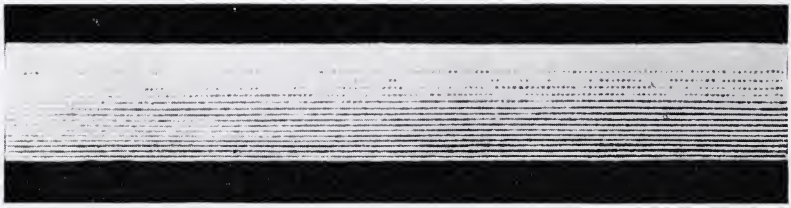


FIG. 10

vibrate, as shown by the character of the image in the revolving mirror, Fig. 9; while, if one of the branches be made half a wave[1] longer than the other, the disturbance disappears, and the image appears as shown in Fig. 10.

A very simple and instructive experiment may be made

[1] The length required will depend on the tone of the organ pipe. For middle C (256 vibrations per second) the double length required is two feet.

by throwing simultaneously two stones into still water, and a number of interesting variations may be obtained by varying the size of the stones and their distance apart.

The experiment may be arranged for projection by using a surface of mercury instead of one of water, and agitating it by means of a tuning-fork, to the ends of whose prongs are attached light pieces of iron wire which dip slightly into the mercury.

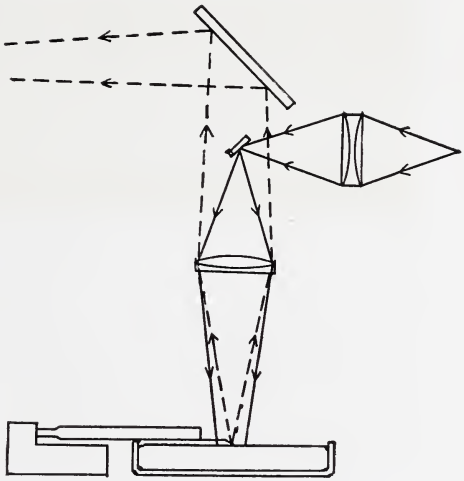


FIG. 11

The arrangement of the apparatus is shown in Fig. 11. The light of an electric lamp is concentrated on a small mirror, by which it is reflected through a lens to the tuning-fork, whose ends dip into a surface of mercury. It is reflected by the mercury surface back through the lens and passes to another mirror, by which it is reflected to form an image on a distant screen. Fig. 12 shows the resulting disturbance of the surface. The circular ripples which diverge from the points of contact of the forks are represented by the circles. These move too rapidly to be seen in the actual experiment, but may be readily recognized in an instantaneous photograph. The heavy lines are the lines of maximum disturbance, where the two systems of waves meet, always in the same phase; while the lighter parts between represent the quiescent portions of the surface, where the crests of one system meet the troughs of the other, forming stationary waves. Fig. 13 is a photograph of the actual appearance.

Another striking instance of interference is furnished by two tuning-forks of nearly the same pitch. Take, first, two similar forks mounted on resonators. When these are sounded by a cello bow, the resultant tone may or may not be louder than the component tones, but it is constant — or, at least, dies away very slowly. If, now, one of the forks be loaded by

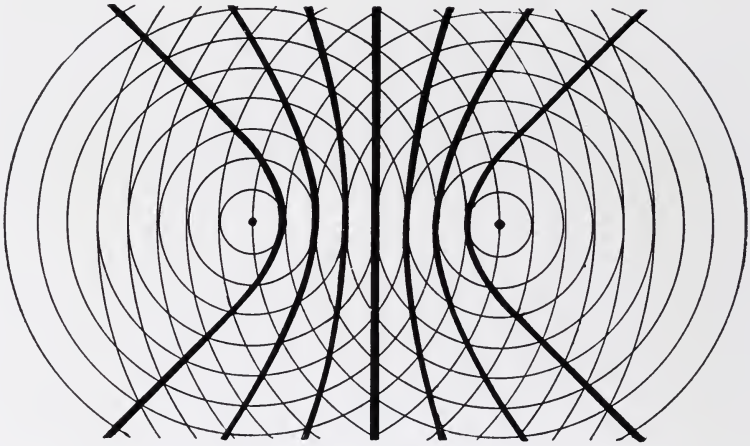


FIG. 12

fastening a small weight to the prong, the sound sinks and swells at regular intervals, producing the well-known phenomenon of "beats." The maximum occurs when the two vibrations are in the same phase. Gradually the loaded fork loses on the other until it is half a vibration behind; then there is a brief silence. This may be shown graphically by allowing each fork to trace its own record along a piece of smoked glass, and by adding the two sine curves, as shown in Fig. 14.

The matter of the interference of light waves requires special treatment on account of the enormous rapidity of the vibrations. This statement, however, inverts the actual chronology, for this rapidity is inferred from the interference experiments themselves.

A beautiful instance of such interference occurs in a soap film. Ordinarily, however, such films have the form

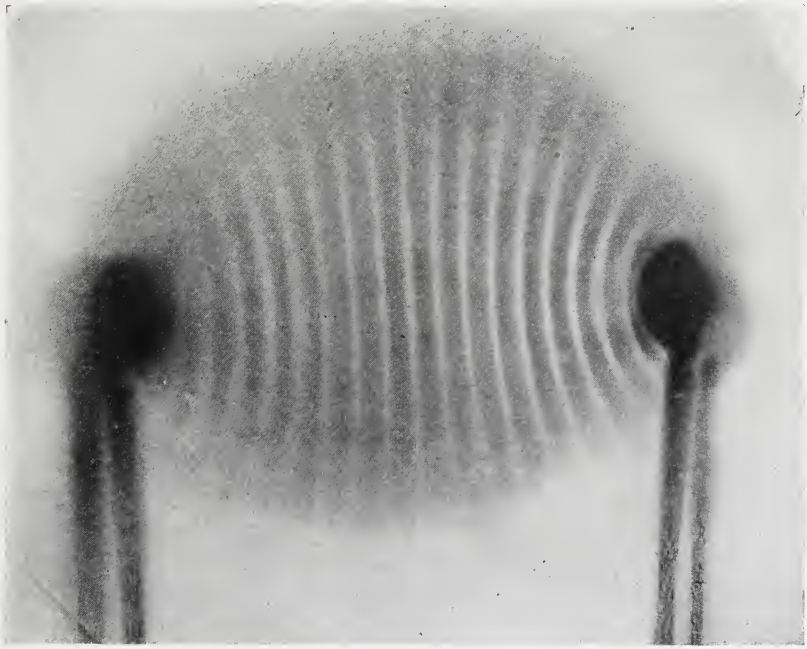


FIG. 13

of a soap bubble; and, while the disturbing causes usually in operation enhance wonderfully the beauty of the appearance, they do not permit the accurate investigation of the

FIG. 14

phenomenon. These disturbing elements are very much diminished in the arrangement which follows:

A soap solution is made up as follows: One part of fresh Castile soap is dissolved in forty parts of warm water; when cool, three parts of the solution are mixed with two parts of glycerine. The mixture is cooled to a temperature of 3° or 4° C., and filtered. A soap film is formed by dipping into the solution a short piece of wide glass tubing. Removing the tube and placing it so that the film is vertical, a series of beautifully colored bands appear, the colors being deeper at the top and gradually fading into barely perceptible alternations of pink and green near the bottom. The bands broaden out as the film gets thinner, but the succession of colors remains the same and may be described as follows: The top of the film is black; then the colors in the first band are bluish gray, white, yellow, and red; those in the second band are, in order, violet, blue, green, yellow, red; the third band is blue, green, yellow, and red; and the succeeding bands green and red. The colors are best observed by using the film as a mirror to reflect the light from a white wall; or the light from a lantern may be reflected to a lens which forms an image of the film on a screen.

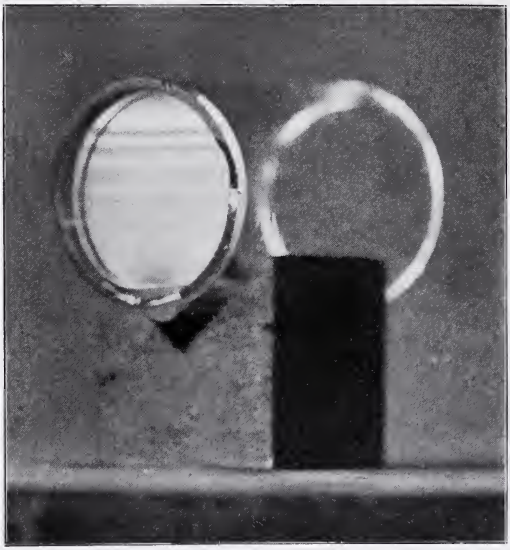


FIG. 15

The colors of thin films and of interference phenomena

generally are among the most beautiful in nature, and while no artist could do justice to such a subject, much less a lithographic plate, such a plate (Plate II) may be used to recall the more striking characteristics.

For the scientific investigation of the interference of light waves, however, the soap film is rather unsatisfactory on account of the excessive mobility of its parts and the resulting changes in thickness. A much more satisfactory arrangement for this purpose is the following: Two pieces of glass with optically plane surfaces are carefully cleaned and freed from dust particles. A single fiber of silk is placed on one of the surfaces near the edge, and the other is pressed against it, thus forming an extremely thin wedge of air between the two plates, as shown in Fig. 16.

FIG. 16

It will be found that in this case the succession of colored bands will resemble in every respect those in the soap film, except that they are now permanent. The light is reflected from all four surfaces, and hence the purity of the colors is somewhat dimmed by the first and the fourth reflections. These may be obviated by using wedges of glass instead of plates.

To account for the colored fringes it will be best to begin with the simpler case of monochromatic light. If a piece of red glass is interposed anywhere in the path of the light, the bands are no longer colored, but are alternately red and black. They are rather more numerous than before, and a trifle wider. If a blue glass is interposed, the bands consist of alternations of blue and black, and are somewhat narrower (*cf.* Plate II).

Let us suppose now that red light consists of waves of very small length. The train of waves reflected by the first surface of the film will be in advance of that reflected by the second surface. At the point where the two surfaces touch

each other the advance is, of course, zero; and here we should have the two wave trains in the same phase, with a consequent maximum of light. Where the thickness of the film is such that the second wave train is half a wave behind, there should be a dark band; at one whole wave retardation, a bright band; and so on.

The alternations of light and dark bands are thus accounted for, but the experiment shows that the first band is dark instead of bright. This discrepancy is due to the assumption that both reflections took place under like conditions, and that the phase of the two trains of waves would be equally affected by the act of reflection. This assumption is wrong, for the first reflection takes place from the *inner* surface of the first glass, while the second occurs at the *outer* surface of the second glass. The first reflection is from a rarer medium — the air; while the second is from a denser medium — the glass. A simple experiment with the Kelvin wave apparatus will illustrate the difference between the two kinds of reflection. The upper end of this apparatus is fixed, while the lower end is free; the fixed end, therefore, represents the surface of a denser medium, the free end that of a rarer medium. If now a wave be started at the lower end by twisting the lowest element to the *right*, the twist travels upward till it reaches the ceiling, whence it returns with a twist to the *left* — *i. e.*, in the *opposite* phase. When, however, this *left* twist reaches the lowest element, it is reflected and returns as a twist to the *left* — so that the reflection is in the *same* phase.

There is thus a difference of phase of one-half a period between the two reflections, and, when this is taken into account, experiment and theory fully agree. We may now make use of the experiment to find a rough approximation to the length of the light waves.

If we measure by the microscope the diameter of the fila-

ment which separates the glasses, it will be found to be, say, two and seven-tenths microns.[1] Counting the number of dark bands in red light, we find there are eight; and hence we conclude that at the thickest part of the air film the retardation is eight waves, and hence the distance separating the glasses — that is, the thickness of the filament — is four waves, which gives about sixty-eight hundredths of

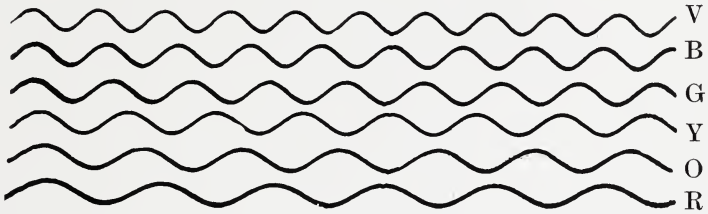


FIG. 17

a micron for the wave length of red light. If blue light is used, there will be twelve dark bands, whence the wave length of blue light is forty-five hundredths of a micron.

The following table gives the approximate wave lengths of the principal colors:

Red - - - -	0.68	microns
Orange - - -	.63	"
Yellow - - -	.58	"
Green - - -	.53	"
Blue - - - -	.48	"
Violet - - -	.43	"

Fig. 17 gives a diagram of the wave lengths of the different colors, magnified about twenty thousand times.

SUMMARY

Waves give information concerning direction, distance, magnitude, and character of the source. Light does the same; hence the presumption in favor of the hypothesis that light consists of waves.

[1] A micron is a thousandth of a millimeter, or about a twenty-five thousandth of an inch.

Wave trains may destroy each other by "interference." Light added to light may produce darkness.

The reason why interference is not more frequently apparent in the case of light is that light waves are exceedingly minute.

By the measurement of interference fringes it is possible to measure the length of light waves, and the results of such measurements show that the wave lengths are different for different colors.

LECTURE II

COMPARISON OF THE MICROSCOPE AND TELESCOPE WITH THE INTERFEROMETER

One of the principal objections which have been urged against the wave theory of light is the fact that light appears to travel in straight lines, whereas sound, which is known to be a wave motion, does not cast a shadow; in other words, the sound waves are capable of bending around an obstacle in the path of the waves.

We shall now not only try to show that both of these two statements are untrue, or, at least, only approximately true, but we shall actually show that sound waves do cast a shadow and that light waves do not move in straight lines. The effect, in fact, depends on the length of the wave, and we may say roughly that the reason why a sound shadow is not ordinarily observed is that the obstacles themselves are of the same order of magnitude as the length of the sound waves. If, therefore, we wish to cast a sound shadow, it will be necessary to use either very large screens or very short waves — that is, high sounds. Indeed, the effect will be most evident if we use sounds that are barely within the limits of audition, or in some cases higher than can be perceived by the ear; and it will be interesting to trace the relation between the definiteness of the sound shadow and the shortness of the sound wave.

I have here a whistle whose length is about one inch. It produces, therefore, a sound wave of the length of four inches. In order to show to an audience the effect of the whistle at different points on the other side of an obstacle, it is convenient to use what is termed a "sensitive flame."

This flame is produced by allowing a jet of gas to issue under considerable pressure from a small nozzle, and by gradually increasing the pressure until the flame is on the point of flaring. On blowing the whistle, we observe that the flame ducks; it is lowered to perhaps one-third or one-fourth of its height, and broadens out at the same time. On placing the whistle behind an obstacle, we observe by the ducking of the flame that it responds to the whistle almost as readily as when no obstacle was present.

I now take a shorter whistle, half an inch long; which, therefore, produces a sound wave two inches long. The flame responds even more readily to this than to the longer whistle, and when the shorter whistle is sounded behind the obstacle the flame ducks, but to a much less marked degree than before.

I have here the means of producing still higher sounds. Strung on a piece of wire are a number of iron washers—rings of iron about an inch in diameter. When these are shaken they produce vibrations whose wave length is even shorter than that produced by the whistle just sounded. On shaking the rings you perceive the immediate response of the flame, and on shaking the rings behind the obstacle the flame responds still, but much more feebly. I take a new set of rings one-half inch in diameter. On shaking these the flame responds as before, but when I place the rings behind the obstacle the flame scarcely responds at all. I take a still smaller series of discs. These are approximately only one-fourth of an inch in diameter and produce a wave whose length is approximately one-half inch. On shaking the last set of discs outside the obstacle the flame responds not quite so strongly as before, because the total amount of energy in this case is very small; but, on shaking the discs behind the obstacle, the flame is absolutely quiescent, showing that the sound shadow is perfect. In moving the discs

to and fro while shaking them, the geometrical limit of the shadow can be definitely marked to within something like half an inch; that is, a quantity of the same order as the length of the sound wave which is being used.

It is evident from the foregoing that, if we wish to investigate the bending of light waves around a shadow, we must take into account the fact which has already been established, namely, that the light waves themselves are exceedingly small — something of the order of a fifty-thousandth of an inch. The corresponding bending around an obstacle might, therefore, be expected to be a quantity of this same order; hence, in order to observe this effect, special means would have to be adopted for magnifying it.

The diffraction of sound waves is beautifully shown by the following experiment:[1] A bird call is sounded about ten feet from a sensitive flame, and a circular disc of glass about a foot in diameter is interposed. If the adjustment is imperfect, the sound waves are completely cut off; but when the centering of the plate is exact, the sound waves are just as efficient as though the obstacle were removed.

This surprising result was first indicated by Poisson, and was considered a very serious objection to the undulatory theory of light. It was naturally considered absurd to say that in the very center of a geometrical shadow there should not only be light, but that the brightness should be fully as great as though no obstacle were present. The experiment was actually tried, however, and abundantly confirmed the remarkable prediction.

The experiment cannot be shown to an audience by projecting on a screen, but an individual need have no difficulty in observing the effect. The image of an arc light (or, better, of the sun) is concentrated on a pinhole in a card, and the light passing through is observed by a lens of two or three

[1] Exhibited by Lord Rayleigh at the Royal Institute.

inches' focal length some twenty feet distant. About half-way a disc of about a quarter-inch diameter, and very smoothly and accurately turned, is suspended by three threads,[1] so that its center is accurately in line with the pin-hole and the center of the lens. The field of the lens will now be quite dark, except *at the center of the shadow*, where a bright point of light is seen.

We shall now attempt to show the analogue of the sound-shadow experiment by means of light waves. The light is

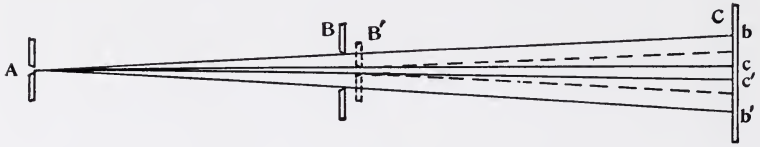


FIG. 18

concentrated on a very narrow slit A (Fig. 18), which may be supposed to act as the source of light waves. Another slit B, about an inch wide, is placed at a distance of about eight feet, and beyond this a screen C receives the light which has passed through B. The borders bb of the shadow of the slit B are quite sharply defined (though a very slight bending of the light around the edges may be observed by means of a lens focused on b). But if the slit be made narrow, as at B', the sharp boundary which should appear at cc is diffuse and colored, the light being bent into the geometrical shadow as indicated by the dotted lines. The narrower the second slit is made, the wider and more diffuse will be the image on the screen; that is to say, the greater will be the amount of bending into the shadow. An interesting variation of the experiment is made by using two slits instead of the second slit B. In this case, in addition to the

[1] The disc may be glued to a piece of *optical* glass, care being taken that no trace of glue appears beyond the edge of the disc.

bending of the rays from their geometrical path, we have the interference of the light from the two slits, producing interference bands whose distance apart is greater the closer the two slits are together. If instead of two slits we have a very large number, such as would be produced by a number of very fine parallel wires, we have, in addition to the central, sharp image, two lateral, colored images, which, when carefully examined, show in their proper order all the colors of the spectrum. This arrangement is known as a *diffraction grating*, and, though mentioned here simply as an instance of diffraction or bending of the rays from their geometrical path, will be shown in a subsequent lecture to have a very important application in spectrum analysis.

We have thus shown that light consists of waves of exceeding minuteness, and have found approximate values of the lengths of the waves by measuring the very small interval between the surfaces at the thicker end of our air wedge. It can be shown also that this same measurement may be accomplished with a grating if we know the small interval between its lines. The question naturally arises: Might it not be advantageous to reverse the process, and, utilizing this extreme minuteness of light waves, make our measurements of length or angle with a correspondingly high order of accuracy? The principal object of these lectures is to illustrate the various means which have been devised for accomplishing this result.

Before entering into these details, however, it may be well to reply to the very natural question: What would be the use of such extreme refinement in the science of measurement? Very briefly and in general terms the answer would be that in this direction the greater part of all future discovery must lie. The more important fundamental laws and facts of physical science have all been discovered, and these

are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote. Nevertheless, it has been found that there are apparent exceptions to most of these laws, and this is particularly true when the observations are pushed to a limit, *i. e.*, whenever the circumstances of experiment are such that extreme cases can be examined. Such examination almost surely leads, not to the overthrow of the law, but to the discovery of other facts and laws whose action produces the apparent exceptions.

As instances of such discoveries, which are in most cases due to the increasing order of accuracy made possible by improvements in measuring instruments, may be mentioned: first, the departure of actual gases from the simple laws of the so-called perfect gas, one of the practical results being the liquefaction of air and all known gases; second, the discovery of the velocity of light by astronomical means, depending on the accuracy of telescopes and of astronomical clocks; third, the determination of distances of stars and the orbits of double stars, which depend on measurements of the order of accuracy of one-tenth of a second—an angle which may be represented as that which a pin's head subtends at a distance of a mile. But perhaps the most striking of such instances are the discovery of a new planet by observations of the small irregularities noticed by Leverier in the motions of the planet Uranus, and the more recent brilliant discovery by Lord Rayleigh of a new element in the atmosphere through the minute but unexplained anomalies found in weighing a given volume of nitrogen. Many other instances might be cited, but these will suffice to justify the statement that "our future discoveries must be looked for in the sixth place of decimals." It follows that every means which facilitates accuracy in measurement is a possible factor in a future discovery, and this will, I trust, be a sufficient excuse for bring-

ing to your notice the various methods and results which form the subject-matter of these lectures.

Before the properties of lenses were known, linear measurements were made by the unaided eye, as they are at present in the greater part of the everyday work of the carpenter or the machinist; though in many cases this is supplemented by the "touch" or "contact" method, which is, in fact, susceptible of a very high degree of accuracy. For angular measurements, or the determination of direction, the sight-tube was employed, which is used today in the alidade and, in modified form, in the gun-sight — a fact which shows that even this comparatively rough means, when properly employed, will give fairly accurate results.

The question then arises whether this accuracy can be increased by sufficiently reducing the size of the apertures.

The answer is: Yes, it can, but only up to a certain limit, beyond which, apart from the diminution in brightness, the diffraction phenomena just described intervene. This limit occurs practically when the diameter of two openings a meter apart has been reduced to about two millimeters, so that the order of accuracy is about $\frac{1}{5} \times \frac{1}{500}$, or $\frac{1}{2500}$, for measurements of angle. Calling ten inches the limit of distinct vision, this means that about $\frac{1}{250}$ of an inch is the limit for linear measurement. An enormous improvement in accuracy is effected by the introduction of the microscope and telescope, the former for linear, the latter for angular measurements. Both depend upon the property of the objective lens of gathering together waves from a point, so that they meet again in a point, thus producing an image.

This is illustrated in Fig. 19. A train of plane waves traveling in the direction of the arrows encounters a convex lens. The velocity is less in glass, and since the lens is

thickest at the center, the retardation is greatest there, gradually diminishing toward the edge. The effect is to change the form of the wave front from a plane to a spherical shell,

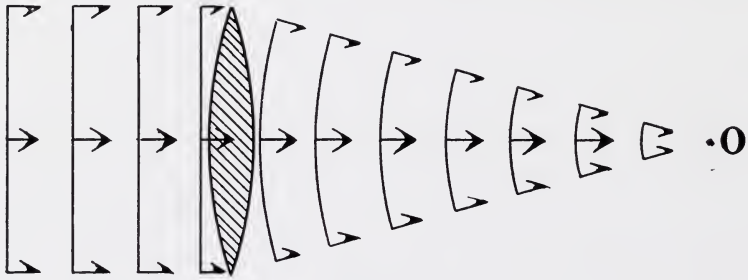


FIG. 19

which advances toward the focus at O, and produces at this point a maximum of light, which is the image of the point whence the waves started.

Fig. 20 illustrates the case where the convex waves diverging from a luminous point O are changed to concave waves converging to form the image at O'.

It can readily be shown that the luminous point and its image are in the same line with the center of the lens—

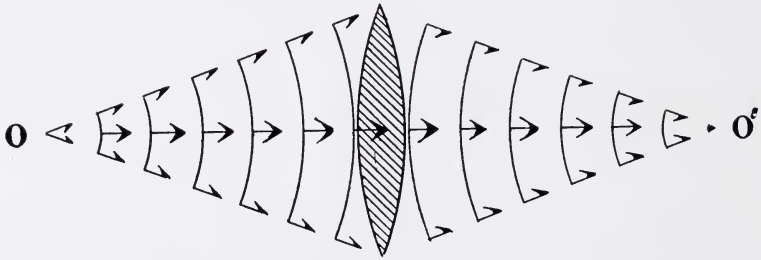


FIG. 20

sufficiently near for a first approximation. Accordingly, if we take separate points of an object, we can construct its image by drawing straight lines from these through the center of the lens, as shown in Fig. 21. The size of the image will be greater the greater the distance from the lens, so that

the magnification is proportional to the ratio of the distances from object and image respectively to the center of the lens; hence in the microscope an error in determining the position of the image means a much smaller error in the determination

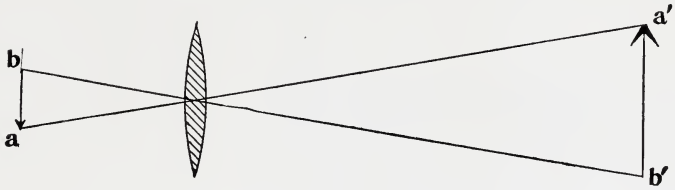


FIG. 21

of the position of the point source. This error could be diminished indefinitely by increasing the magnifying power, were it not for the attendant loss of light and the fact that the light waves, though very minute, are not infinitesimally small. In fact, the same diffraction effects again limit the possibility of indefinite accuracy of measurement. What, then, is the new limit?

Let p, Fig. 22, represent the center of the geometrical

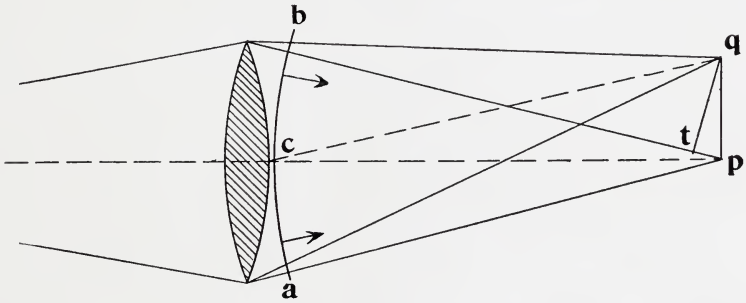


FIG. 22

image of a luminous point. This will be a point of maximum brightness, because all parts of the concave wave which converges toward p reach this point at the same time, and therefore in the same phase. Let us consider an adjacent point q. The parts of the converging wave are no longer at equal

distances from this point, and hence will not arrive in the same phase, and the brightness will be less than at p. At a certain distance pq there will be no light at all. This occurs when the difference of phase between the extreme ray and the central ray is half a wave, that is, calling the wave length l, when $cq - bq = \frac{1}{2} l$; for these two pairs of rays destroy each other, and the same is true of every two such pairs of rays.

The same is equally true of every point about p at this same distance; hence there will be a dark ring about the bright image. This is succeeded by a bright ring, a second dark ring, and so on.

The radius of the first dark ring may be calculated as follows:

Draw qt at right angles to bp. Then $cq - bq = \frac{1}{2} l$. But $cq = cp$, very nearly, and $cp = bp$, and $bq = bt$, so that $bp - bq = pt = \frac{1}{2} l$.

But the triangles pqt and pbc are similar, whence $pt : pq = bc : bp$; or, calling r the radius of the first dark ring, F the focal length of the lens, and D the diameter of the lens, $r = \frac{F}{D} l$; that is, the radius of the dark ring is greater than the length of the light wave, in the same proportion as the focal length of the lens is greater than its diameter.[1] For example, if the length of the light wave be taken as one fifty-thousandth of an inch, and the focal length of the lens as one hundred times the diameter, then this radius will be one five-hundredth of an inch — a quantity readily perceptible with a moderate eyepiece. The lack of distinctness of the image would be of the same order, and would be further aggravated by greater magnification, resembling a drawing made with a blunt point.

[1] Strictly, this is about one-fourth greater on account of the fact that the aperture is circular instead of rectangular.

In most cases these diffraction rings are so small that they escape notice, unless the air is unusually quiet and the lens exceptionally good. If these conditions are satisfied, and the instrument is focused on a very small or distant bright object (a star, or a pinhole in front of an electric arc), the rings are readily visible with a sufficiently high-power eye-piece. They may be much more readily observed, however, if the ratio of diameter to focal length be diminished by placing a circular aperture before the lens. The smaller the aperture, the larger will be the diffraction rings. Fig. 23 is a photograph of the phenomenon, showing the appearance of the rings when the diameter of a lens of five meters' focal length has been reduced to one centimeter.

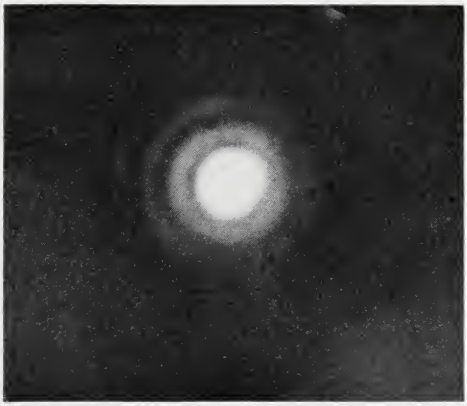


FIG. 23

In the case of a telescope the corresponding limiting angle is the angle subtended by r at the distance F, *i. e.*, $\frac{r}{F}$, and this, by the formula, is the same as the angle subtended by the light wave at the distance D — the diameter of the objective. This limiting angle for a five-inch lens would, therefore, be $\frac{1}{250000}$ of an inch, *i. e.*, about the size of a quarter of a dollar viewed at the distance of a mile. This could be measured to within one-fifth of its value, so that the accuracy of measurement in this case corresponds to $\frac{1}{1250000}$ as against $\frac{1}{2500}$ without the lens; *i. e.*, the order of accuracy is increased about five hundred times.

For a microscope it will be simpler to proceed a little differently. The magnification increases as the object approaches the front of the objective lens. Suppose it is almost in contact. The waves from p (Fig. 24) reach o in the same phase, but those from q reach o more quickly through the upper half of the lens than through the lower half. Let the difference in the paths qao and qbo be l, that is, one of the light waves. Then there will be darkness at o so far as the

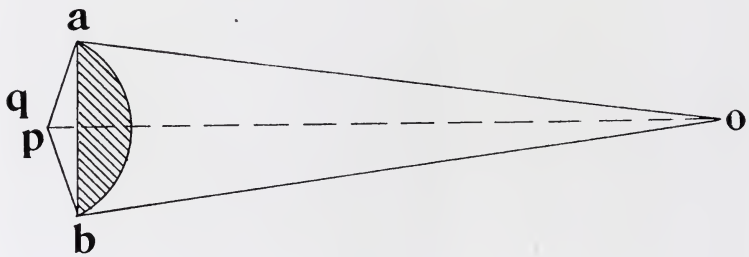


FIG. 24

point q is concerned; *i. e.*, the dark ring in the image of q will lie at o and will thus coincide with the bright center of the image of p. This condition of affairs corresponds to a displacement $pq = \frac{1}{2}l$. Hence, if there were two luminous points at a distance $pq = \frac{1}{2}l$ apart, their diffraction images would overlap so as to be indistinguishable from each other. Hence $\frac{1}{2}l$, or $\frac{1}{100000}$ of an inch, is the "limit of resolution" in any microscope, as against $\frac{1}{250}$ of an inch with the naked eye. So that here again the increase in accuracy is about four hundred times.

These theoretical deductions are amply confirmed by actual observation, and since in this investigation we have supposed a theoretically perfect lens, these results show that our present microscopes and telescopes, when operated under proper conditions, are almost perfect instruments.

Thus, Fig. 25 shows a micro-photograph of the specimen called *Amphipleura pellucida*, whose markings are about

100,000 to the inch. This is about the theoretical limit for blue light. By using the portion of the spectrum beyond the violet it might be possible to go still farther.

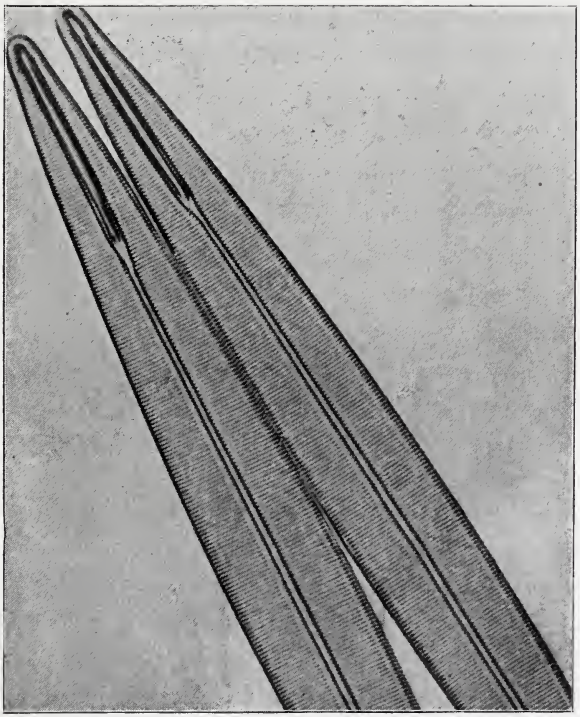


FIG. 25

Doubtless by a much higher magnification a much more accurate setting on a given phase of the fringes could be made, and hence a corresponding increase of accuracy of measurement could be attained. But this involves a great loss of light, since the intensity varies inversely as the square of the magnification. Consequently, even with a threefold magnification the intensity is diminished ninefold, so that it would be difficult to see the image unless the illumi-

nation were so powerful as to endanger the specimen, or to introduce temperature variations which would vitiate the results of the measurement.

It is apparent from all that precedes that in all measurements by the microscope or the telescope we are, in fact,

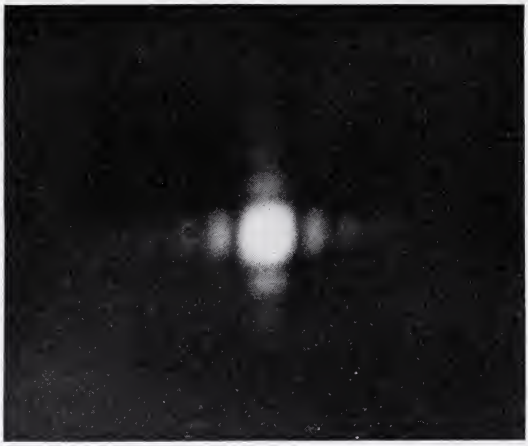


FIG. 26

making use of the interference of light waves. Let us see, then, if we are making the best use of this interference, or whether it may not be possible to increase the high degree of accuracy already attained.

It has just been shown that, in the case of a telescope, the angular magnitude of the diffraction rings, and with this the accuracy of measurement of the position of the luminous point, depends only on the diameter of the objective. Now, the form of the fringes will of course vary with the form of the aperture, and if this be square instead of circular, the diffraction image will be represented by Fig. 26, which may be compared with Fig. 23. The width of the fringes is but little altered, while there is a perceptible increase in dis-

tinctness. Let the middle part of the aperture now be covered up, as in Fig. 27, so that the light can pass through the uncovered portions, *a* and *b*, only. Fig. 28 shows the appearance of the fringes in this case. The distribution is somewhat different, but the distinctness is considerably increased, so that the position of the center of any fringe (the central bright fringe, for instance) may be measured with a decided increase in accuracy. The utilization of the two portions of a lens, at opposite ends of a diameter, converts the telescope or microscope into an *interferometer*.

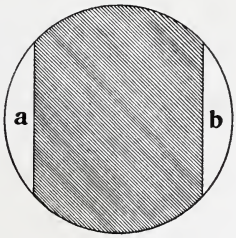


FIG. 27

This term is used to denote any arrangement which separates a beam of light into two parts and allows them to reunite under conditions to produce interference. The path of the separated pencils may be varied in every possible way; for instance, by interposing prisms or mirrors, provided the optical paths are nearly equal and the angle between the two final directions very small. The first condition is essential only when the light is not homogeneous. The reason will be apparent when it is remembered that the width of the interference bands depends on the wave length of the light employed. If the light is composite, as in the case of white light, each component will form interference bands whose width is proportional to the wave length.

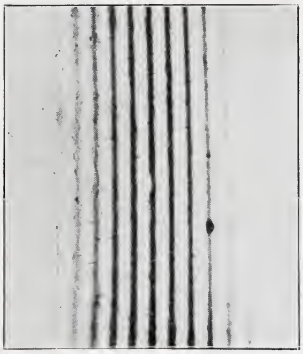


FIG. 28

This is illustrated in Fig. 29, where the fringes due to red, yellow, and blue light respectively are separated. In

the actual experiment, however, they are all superposed. At the middle point, where the two paths are equal, all the colors will be superposed, the result being a *white* central band. At no other point will this be true, and the result will be a series of colored fringes symmetrically disposed about the central white fringe, the succession of colors being exactly the same as in the case of thin films (*cf.* Plate II).

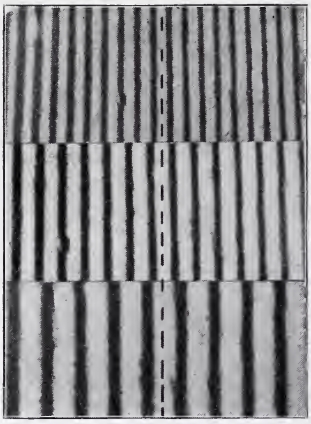


FIG. 29

The breadth of the fringes is determined by the smallness of the angle under which the two pencils meet. This is shown in Fig. 30. In the right-hand figure the angle between the pencils is smaller than in the other, while the breadth of the fringes is correspondingly greater in the former than in the latter. The exact

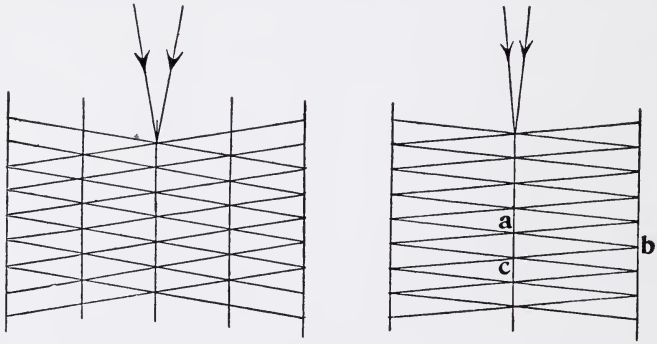


FIG. 30

relation is readily obtained. We have only to note that ac is the wave length l (very nearly) and bc is (very nearly) the width b of a fringe; whence, if e is the very minute angle

at b (which is the same as the angle between the directions of the interfering pencils), $b = \frac{l}{e}$; or, in other words, the width of the fringes is proportional to the wave length of the light, and inversely proportional to the angle between the pencils.

Thus, if the pencils converge from two apertures a quarter of an inch apart, and meet at a screen ten feet away, the breadth of the fringes will be one-hundredth of an inch.

The importance of using a very small angle will be noted.

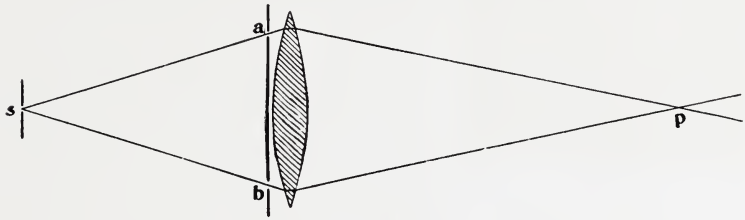


FIG. 31

In this simple form of interferometer the angle can be made small only by bringing the two apertures very near together, which seriously diminishes the efficiency of the instrument; or by increasing the distance from the openings to the fringes, or by using a high magnification, which enfeebles the light, already very faint in consequence of having to start from a pinhole or a narrow slit s (Fig. 31) and to pass through the narrow apertures a and b. There is, therefore, but little advantage in this form of interferometer over the corresponding older analogues (microscope and telescope).

An important improvement may be effected by bending one or both the rays ap, bp by reflections in such a way as to diminish the angle at p, as shown in Fig. 32.

A further improvement is effected by replacing the apertures a and b by mirrors; and, finally, by replacing the slit

s by a plane surface. The interferometer is now changed into the form illustrated in Fig. 33. It will now be noted that the source need no longer be a point or a slit, but may be a broad flame; and the object whose position is to

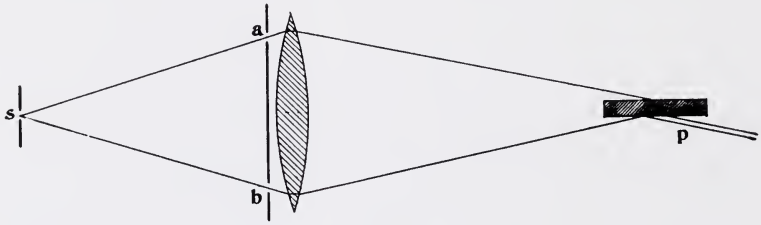


FIG. 32

be measured is no longer a fine line or a slit, but a flat surface. The width of the fringes may be made as great as we please without any sacrifice in the brightness of the light. The corresponding increase in accuracy is from twenty to one hundred fold. We may conveniently restrict the term *interferometer* to this arrangement, in which the division and the union of the pencils of light are effected by a transparent plane parallel plate. It is important to note that the path of the two pen-

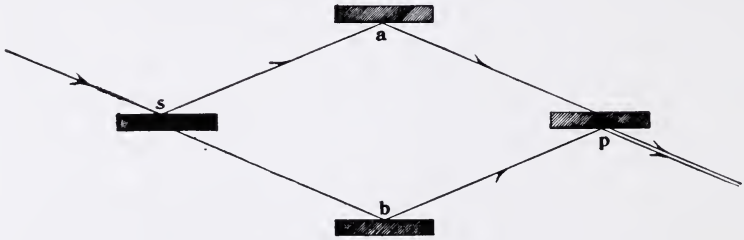


FIG. 33

cils after their separation by the first plate is entirely immaterial; for example, either or both pencils may suffer any number of reflections or refractions before they are reunited by the second plate, without affecting in any essential point the efficiency of the interferometer, provided that the differ-

ence in the path of the two pencils is not too great, and provided that the two pencils are reunited at a sufficiently small angle. By altering these conditions of reflection or

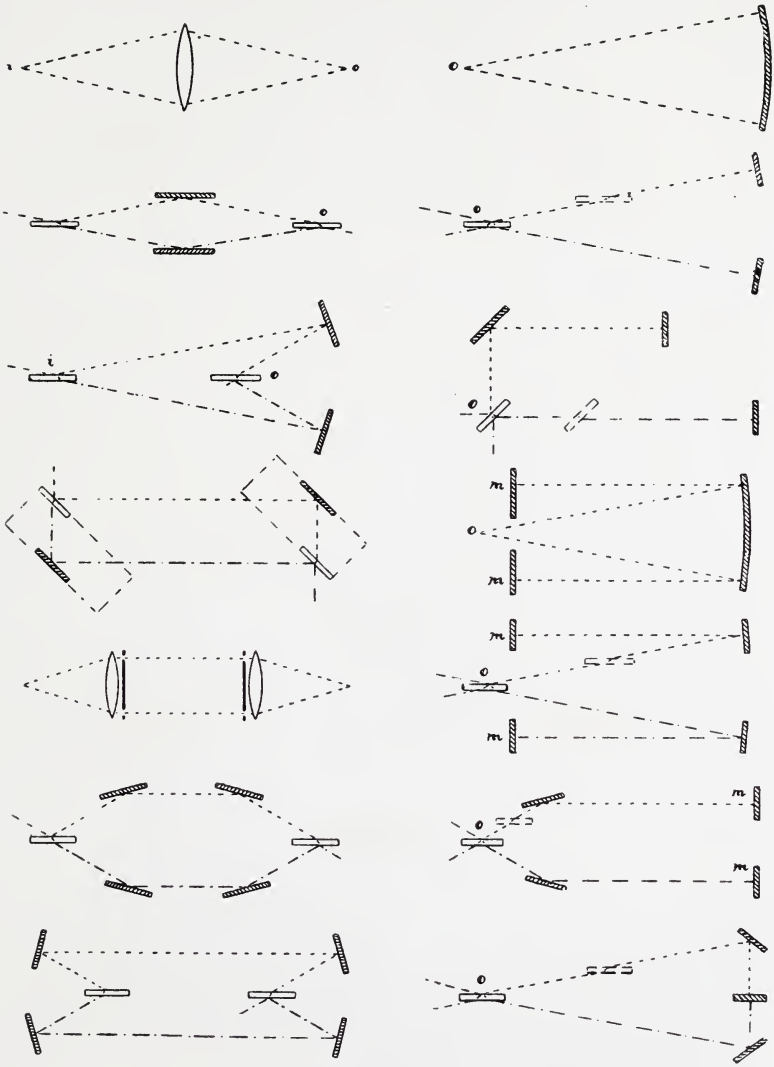
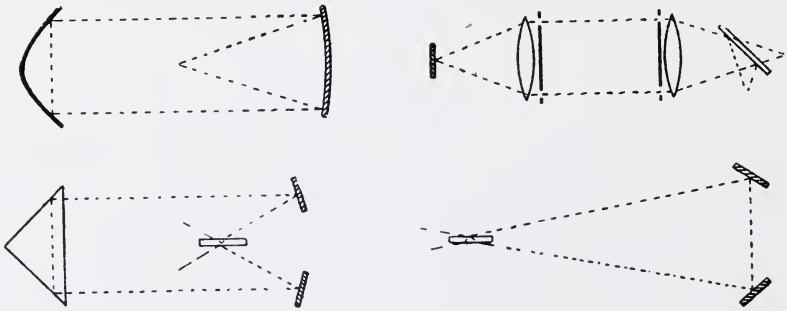


FIG. 34

refraction we may obtain a very considerable number of variations of form, as illustrated in Figs. 34, 35.

One of these types, enlarged in Fig. 36, has been arranged



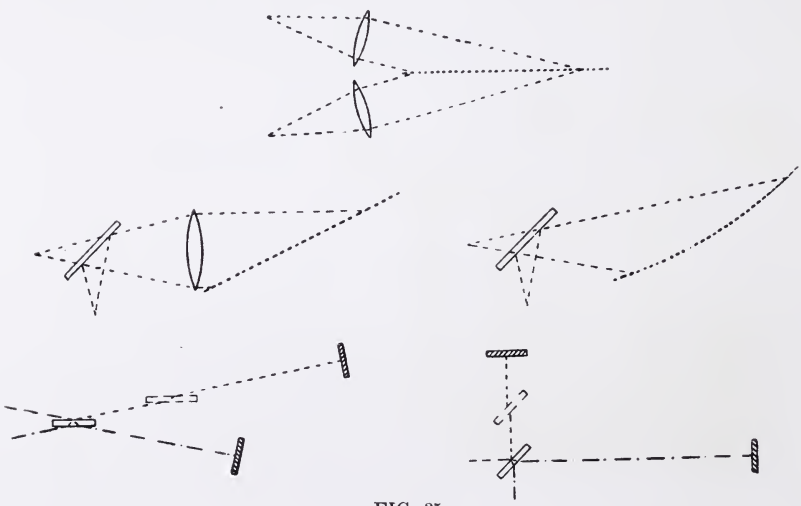


FIG. 35

in such a way as to show the extreme delicacy of the interferometer in measuring exceedingly small angles. For this purpose two of the mirrors, *C* and *D*, have been mounted on a piece of steel shafting *P* two inches in diameter and six inches long. When the length of the paths of the two pencils is the same to within a few hundred thousandths of an inch, the interference fringes in white light are readily observed, or may be projected on the screen. If, now, the steel shafting be twisted, one of the paths is lengthened and the other diminished, and for every movement of one two-hundred-thousandth of an inch there would be a motion of the fringes equal to the width of a fringe. Now, taking the end of the steel shafting between thumb and forefinger, the exceedingly small force which may thus be applied in this way is sufficient to twist

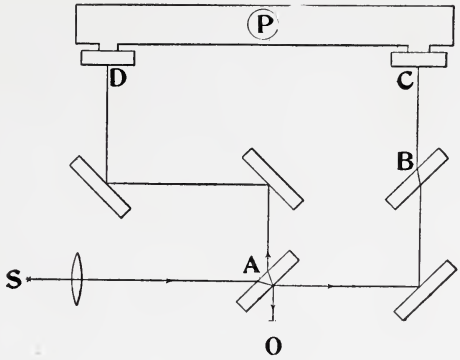


FIG. 36

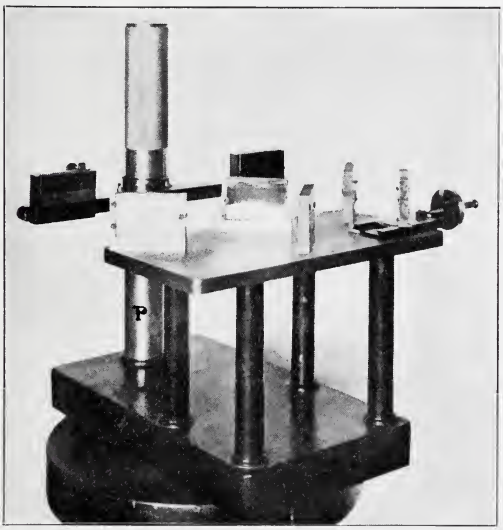


FIG. 37

the solid steel shafting through an angle which is very readily observed by the movement of the fringes across the field.

The form of interferometer which has proved most generally useful is that shown in Fig. 38. The light starts from source S and separates at the rear of the plate A, part of it being reflected to the plane mirror C, returning exactly, on its path through A, to O, where it may be examined by a telescope or received upon a screen. The other part of the ray goes through the glass plate A, passes through B, and is reflected by the plane mirror D, returns on its path to the starting-point A, where it is reflected so as nearly to coincide with the first ray. The plane-parallel glass B is introduced to compensate for the extra thickness of glass which the first portion of the ray has traversed in passing twice through the plate A. Without it the two paths would not be optically identical, because the first would contain more glass than the second.

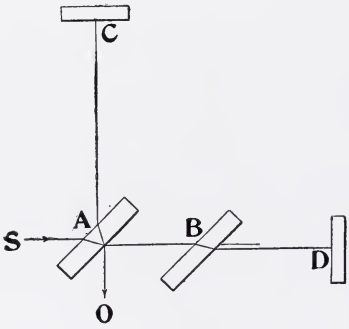


FIG. 38

Some light is reflected from the front surface of the plate A, but its effect may be rendered insignificant by covering the rear surface of A with a coating of silver of such thickness that about equal portions of the incident light are reflected and transmitted.

The plane-parallel plates A and B are worked originally in a single piece, which is afterward cut in two. The two pieces are placed parallel to each other, thus insuring exact equality in the two optical paths AC and AD.

The foregoing principles are applied in concrete form in the instrument shown in Figs. 39, 40. A rigid casting serves

as the bed of the instrument. One end of this bed has fastened to it a heavy metal plate *H*, which carries the three glass plates *A*, *D*, and *B*. The plate *A* is held in a metal frame which is rigidly fastened to the plate *H*. The frame which holds *B* can be turned slightly about a vertical axis to allow of adjusting *B* so that it is parallel to *A*. The mirror *D* is held by springs against three adjusting screws which are set in a vertical plate attached to the end of the plate *H*. Both *C* and *D* are silvered on their front faces. The frame which holds the mirror *C* is firmly mounted on a metal slide which can be moved by the screw *S* along the ways *EF*. One very essential feature of the apparatus is that these ways shall be so true that the mirror *C* shall remain parallel to itself as it is moved along. The accuracy of the ways must be so great that the greatest angle through which the mirror *C* turns in passing along them is less than one second of an arc. This accuracy cannot be attained by the instrument maker, but the final grinding must be done by the investigator himself.

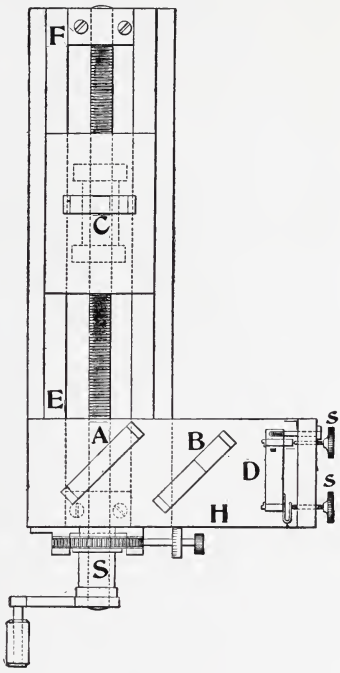


FIG. 39

To adjust the instrument so that fringes are formed, a small object like a pin is held between the source and the plate *A*. Two images of this pin will be seen by an observer at *O*—one formed by the light which is reflected from *C*, and the other by that reflected from *D*. The fringes in

monochromatic light will appear when these two images have been made to coincide with the help of the adjusting screws *ss*. The fringes in white light appear only when the lengths of the two paths AD and AC are the same. The

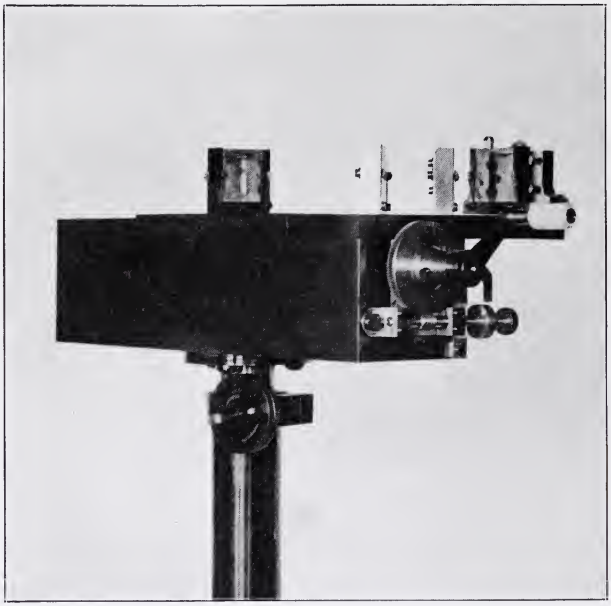


FIG. 40

width and the position of the fringes in the field of view can be varied by slightly moving the adjusting screws. We shall have occasion to discuss this particular form of interferometer in a subsequent lecture.

SUMMARY

1. The objection to the wave theory of light, that light moves in straight lines while sound waves can bend around an obstacle, is shown to be groundless, since we have seen that if the sound waves are sufficiently short they cast a sound shadow, while by devices which take into account the

extreme minuteness of light waves their bending around obstacles may be readily observed.

2. The extreme minuteness of light waves renders it possible to utilize the microscope and the telescope as instruments of great precision. These instruments depend on the property of the objective of gathering together waves from a point so that they are concentrated in the diffraction pattern which is called the image.

3. The accuracy of measurement is still further increased by modifying the telescope or microscope so as to utilize only two pencils, thus converting these instruments into interferometers.

4. By the device of separating the two pencils and reuniting them by reflections from plane-parallel surfaces, the fringes may be made as large as we please without diminishing the brightness of the light, and hence the accuracy of measurement may be correspondingly increased.

LECTURE III

APPLICATION OF INTERFERENCE METHODS TO MEASUREMENTS OF DISTANCES AND ANGLES

In the last lecture we considered the limitations of the telescope and microscope when used as measuring instruments, and showed how they may be transformed so that the diffraction and interference fringes which place the limit upon their resolving power may be made use of to increase the accuracy of measurements of length and of angle. We have named these new forms of instrument interferometers and illustrated many of the forms in which they may be made.

It has been found that the particular form of interferometer described on p. 40 is the most generally useful, and the principal subject of this lecture will be to illustrate the applications which have already been made of this instrument.

But before passing to the first application of the interferometer, we may make a little digression, and consider briefly the two theories which have been proposed to account for the various phenomena of light. One of these is the undulatory theory, which has already been explained; the other is the corpuscular theory, which for a long time held its ground against the undulatory theory, principally in consequence of the support of Newton.

The corpuscular theory supposes that a luminous body shines in virtue of the emission of minute particles. These corpuscles are shot out in all directions, and are supposed to produce the sensation of vision when they strike the retina. The corpuscular theory was for a long time felt to be unsatisfactory because, whenever a new fact regarding light was discovered, it was always necessary to make some supplemen-

tary hypothesis to strengthen the theory; whereas the undulatory theory was competent to explain everything without the addition of extra hypotheses. Nevertheless, Newton objected to the undulatory theory on the ground that it was difficult to conceive that a medium which offers no resistance to the motion of the planets could propagate vibrations which are transverse (and we know that the light vibrations are transverse because of the phenomena of polarization), for such vibrations can be propagated only in a medium which has the properties of a solid. Thus, if the end of a metal rod be twisted, the twist travels along from one end to the other with considerable velocity. If the rod were made of sealing wax, the twist would rapidly subside. If such a rod could be made of liquid, it would offer virtually no elastic resistance to such a twist.

Notwithstanding this, the medium which propagates light waves, and which was supposed to resist after the fashion of an elastic solid, must offer no appreciable resistance to such enormous velocities as those of the planets revolving in their orbits around the sun. The earth, for example, moves with a velocity of something like twenty miles in a second, has been moving at that rate for millions of years, and yet, as far as we know, there is no considerable increase in the length of the year, such as would result if it moved in a resisting medium. There are other heavenly bodies far less dense than the earth, *e. g.*, the comets, and it seems almost incredible that such enormously extended bodies with such an exceedingly small mass should not meet with some resistance in passing through their enormous orbits. The result of such resistance would be an increase in the period of revolution of the comets, and no such increase has been detected. We are thus required to postulate a medium far more solid than steel and far less viscous than the lightest known gas.

These two suppositions are possibly not as inconsistent as they may at first seem to be, for we have a very important analogy to guide us. Consider, for example, shoemaker's wax, or pitch, or asphaltum. These substances at ordinary temperatures are hard, brittle solids. If you drop them, they break into a thousand pieces; if you strike them (so lightly that they do not break), they emit a sound which corresponds to the transverse vibrations of a solid. If, however, we place one of these substances on an inclined surface, it will gradually flow down the incline like a liquid. Or if we support a cake of shoemaker's wax on corks and place bullets on its upper surface, after a time the bullets will have sunk to the bottom, and the corks will be found floating on top. So in these cases we have a gross and imperfect illustration of the co-existence of apparently inconsistent properties such as are required in our hypothetical medium.[1] Nevertheless, it seemed impossible to Newton to conceive a medium with such incompatible properties, and this was, as stated above, a serious obstacle in the way of his accepting the undulatory theory. There were others, which need not now be mentioned.

For a long time after the various modifications that the corpuscular theory had to receive had been made, both theories were actually capable of explaining all the phenomena then known, and it seemed impossible to decide between them until it was pointed out that the corpuscular theory made it necessary to suppose that light traveled faster in a denser medium, such as water or glass, than it does in a rarer medium, such as air; while according to the undulatory theory the case is reversed. We may illustrate briefly the two cases: No matter what theory we accept, it is an observed fact that refraction takes place when light passes

[1] The specialization of the undulatory theory known as the electro-magnetic theory does not remove this difficulty; for it is even more difficult to account for the properties of a medium which is the seat of electric and magnetic forces.

from a denser to a rarer medium, and consists in a bending of the incident ray toward the normal to the surface of the denser medium. Suppose we have a plate of glass, for example, and a ray of light falling upon the surface in any direction. According to the corpuscular theory, the substance below the surface exerts an attraction upon the light corpuscles. Such attraction can act only in the direction of the normal. If we separate it into two components, one in the surface and one normal to it, the normal one will be increased. These two components might be represented by *OA* and *OB* in Fig. 41, and the resultant of the two would be *OC*. In consequence of the presence of the denser medium, the normal component of the velocity of the particle is increased, and the resultant is now *OC′*, which is greater than *OC*.

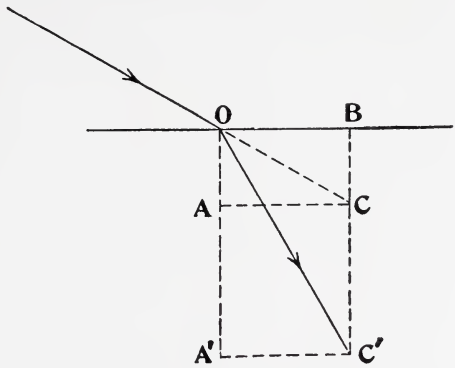


FIG. 41

Let us next consider refraction according to the wave theory. A wave front *ab* (Fig. 42) is approaching the surface *ac* of a denser medium in the direction *bc*. This direction is changed by refraction to *ce*, and the corresponding direction of the new wave front is *cd*. During the time that the wave *ab* moves through the distance *bc* in the rarer medium, it moves through the *smaller* distance *ad* in the denser. Thus the results, according to the two theories, are exactly reversed.

Hence, if we could measure the enormous speed of light—about 400,000 times as great as that of a rifle bullet—it would be possible to put the two theories to the test. In order to

accomplish this we must compare the velocities of light in air and in some denser, transparent medium—say water. Now, the greatest length of a column of water which still permits enough light to pass to enable us to measure the very small quantities involved is something like thirty feet. We should therefore have to determine the time it takes the light to pass through thirty feet of water, at the rate of 150,000 miles a second. This interval of time is of the order of one twenty-millionth of a second. But we must measure a time interval even smaller than this, for we have to distinguish between the velocity in water and the corresponding velocity in the air, *i. e.*, to determine the difference between two time intervals, each of which is of the order of one twenty-millionth of a second. This, at first sight, seems beyond the possibility of any physical experiment; but, notwithstanding this exceedingly small interval of time, by the combined genius of Wheatstone, Arago, Foucault, and Fizeau the problem has been successfully solved. The method proposed by Wheatstone for measuring the velocity of electricity was this: A mirror was mounted so that it could be revolved about an axis parallel to its surface at a very high rate, and the light from the spark produced by the discharge of a condenser was allowed to fall on the mirror. The images of two sparks were observed in the revolving mirror; the second spark passed after the electric current which produced it had passed through a considerable length of wire—

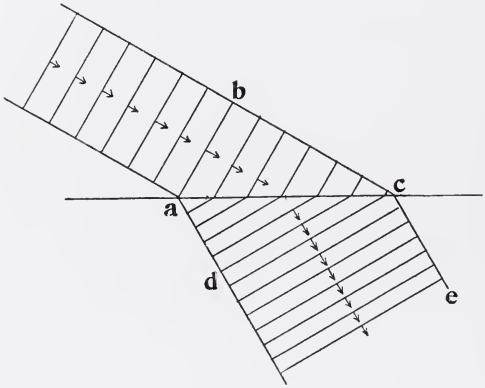


FIG. 42

perhaps several miles; the first, after it had passed through only a few feet of wire. If the mirror in this interval had turned through a perceptible angle, the reflected light would have moved through double that angle; and, knowing the velocity of rotation of the mirror, and measuring this small angle, the velocity of electricity could be determined. Arago thought this same method might be adapted to the measurement of the velocity of light.

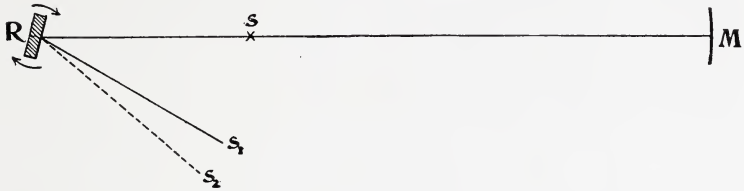


FIG. 43

The principle of Arago's method may be illustrated as follows: Suppose we have a mirror R (Fig. 43), revolving in the direction of the arrows. s is a spark from a condenser, which sends light directly to the mirror R, and also to the distant mirror M, whence it returns to R, and both rays are reflected in the direction s_1. If, however, the light takes an appreciable time to pass from s to M and back, this light will reach the mirror R later, and the mirror will have turned in the interval so as to reflect the light to s_2.

If the angle s_1Rs_2 can be measured, the angle through which the mirror moves is one-half as great; and, knowing the speed of the mirror, we know also the time it takes to turn through this angle; and this is the time required for light to traverse twice the distance sM, whence the velocity of light.

The principle of Arago's method is sound, but it would be extremely difficult to carry it into practice without an important modification, due to Foucault, which is illustrated

in Fig. 44. Light from a source s falls on the revolving mirror R, and by means of a lens L forms an image of s at the surface of a large concave mirror M. The light retraces its path and forms an image which coincides with s if the mirror R is at rest or is turning slowly. When the rotation is sufficiently rapid the image is formed at s_1, and the displacement ss_1 is readily measured.

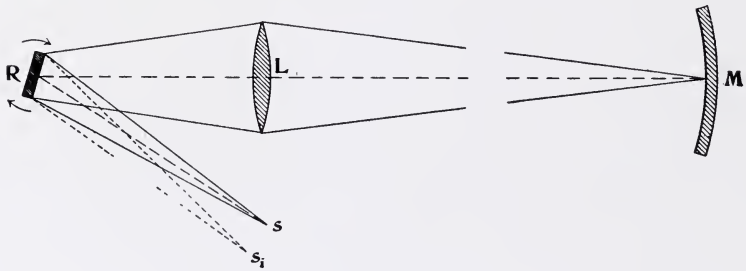


FIG. 44

If the distance LM is occupied by a column of water, the displacement would be less if the velocity of light is greater in water than in air, as it should be according to the corpuscular theory; and if the undulatory theory is correct, the displacement would be greater. Foucault found the displacement greater, and thus the corpuscular theory received its death-blow.

It remained for subsequent experiment to determine whether the undulatory theory was true, because it was not sufficient to show that the velocity was smaller in water; it was necessary to show that the ratio of the two velocities was equal to the *index of refraction* of the water, which is 1.33. Experiments showed that the ratio of the two velocities is almost identical with this number, thus furnishing an important confirmation of the undulatory theory.

Ordinarily the index of refraction is found by measuring the amount of bending which a beam of light experiences in

passing from air into the medium in question. But if this number is identical with the ratio of the velocities, the index would evidently be determined if we knew the ratio of the wave lengths, since the wave lengths are also proportional to the velocities. This can be obtained by the interferometer. In fact, the original name of the instrument is "interferential refractometer," because it was first used for this purpose by Fresnel and Arago in 1816. This name, however, is as cumbersome as it is inappropriate, for, as we shall see, the range of usefulness of the instrument is by no means limited to this sort of measurement.

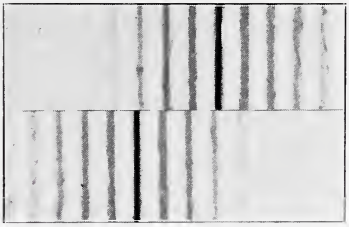


FIG. 45

The interferometer being adjusted for white light, the colored interference fringes are thrown on the screen. If, now, the number of waves in one of the paths be altered by interposing a piece of glass, the adjustment will be disturbed and the fringes will disappear; for the difference of path thus introduced is several hundreds or thousands of waves; and, as shown in the preceding lecture, the fringes appear in white light only when the difference of path is very small.

The exact number of waves introduced can readily be shown to be $2(n-1)\frac{t}{l}$; that is, twice the product of the index less unity by the thickness of the glass divided by the length of the light wave. Thus, if the index of the glass plate is one and one-half and its thickness one millimeter, and the wave length one-half micron, the difference in path would be two thousand waves.

Let us take, therefore, an extremely thin piece of mica, or a glass film such as may be obtained by blowing a

bubble of glass till it bursts. Covering only half the field with the film, the fringes on the corresponding side are shifted in position, as shown in Fig. 45, and the number of fringes in the shift is the number of waves in the difference of path, from which the index can be calculated by the formula.[1]

FIG. 46

The interferometer is particularly well adapted for showing very slight differences in the paths of the two interfering pencils, such, for instance, as are produced by inequalities in the temperature of the air. The heat of the hand held near one of the paths is quite sufficient to cause a wavering of the fringes; and a lighted match produces contortions such as are shown in Fig. 46. The effect is due to the fact that the density of the air varies with the temperature; when the air is hot its density diminishes, and with it the refractive index.

It follows that, if such an experiment were tried under proper conditions, so that the displacement of the interference fringes were regular and could be measured—which means that the temperature is uniform throughout—then the movement of the fringes would be an indication of temperature. Comparatively recently this method has been used to measure very high temperatures, such as exist in the interior of blast furnaces, etc.

In one of the preceding lectures an image of a soap film was thrown on the screen, and it was shown that the thickness of the film increased regularly from top to bottom, and that where the thickness was sufficiently small the interference fringes enable us to deduce the thickness of the

[1] For quantitative measurements it is necessary to employ monochromatic light. The shifting of the central band of the colored fringes in white light does not give even an approximately accurate result.

film. It was also shown that at the top of the film, where the thickness was very small, a black band appears, its lower edge being sharply defined as though there were here a sudden change in thickness, as illustrated in Fig. 47.

Now, this "black spot" may be observed sufficiently long to measure the displacement produced in interference fringes when the film is placed in the interferometer. It is probable that over the area of the "black spot" the two surfaces of the film are as near together as possible; and if the water is made up of molecules, there are very few molecules in this thickness — possibly only two — so that a measurement of this thickness would give at least an upper limit to the distance between the molecules.

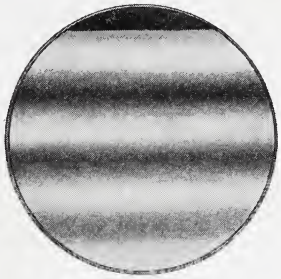


FIG. 47

A soap solution of slightly different character from that used in the last lecture is more serviceable for this purpose.[1] With such a solution the film lasts a remarkably long time. It is interesting to note that some time after the "black spot" has formed, portions of its surface reflect even less light than the rest, and these portions gradually increase in size and number till the whole surface almost entirely vanishes.

It is found on placing such a film as this in the interferometer that there is no appreciable change in the fringes. The film is so thin that we cannot observe any displacement at all; if we place two films in the interferometer, the displacement should be twice as great; but even then it is inappreciable. To obtain a measurable displacement it was found necessary to use fifty such films. The arrangement

[1] This solution is made of caustic soda 1 gm., oleic acid 7 gm., dissolved in 600 c.c. of water.

of the interferometer for this experiment[1] is shown in Fig. 48. The films are introduced in the path AC, as indicated at F. Yet even fifty films produced a displacement of only about half a fringe, as shown in Fig. 49. Since the light passed through each film twice, this displacement of half a fringe is what would be produced by a single passage through one hundred films. One film would therefore produce a displacement of one two-hundredths of a fringe. A simple calculation tells us that the corresponding distance between the water molecules is not greater than six millionths of a millimeter. It may be much less than this.

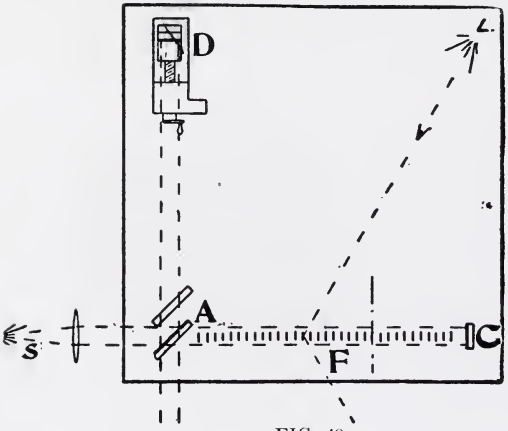


FIG. 48

The interferometer is especially useful whenever it is necessary to measure small changes in distance or angle. One rather important instance of such a measure is that of coefficient expansion. Most bodies expand with heat—certainly a very small quantity: one or two parts in ten thousand for a change of temperature of a single degree.

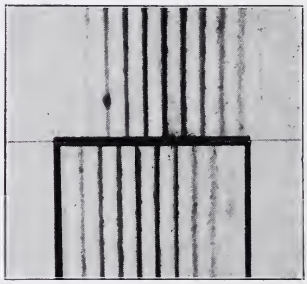


FIG. 49

In some cases it may be necessary to experiment upon a very small specimen of the material in question, and in such cases the whole change to be measured may be of the order of a ten-thousandth part of an inch—

[1] E. S. JOHONNOTT, *Phil. Mag.* (5), Vol. XLVII (1899), p. 501.

a quantity requiring a good microscope to perceive; but such a quantity is very readily measured by the interferometer. It means a displacement amounting to several fringes, and this displacement may be measured to within a fiftieth of a fringe or less; so that the whole displacement may be measured to within a fraction of 1 per cent. Of course, with long bars the attainable degree of accuracy is far greater.

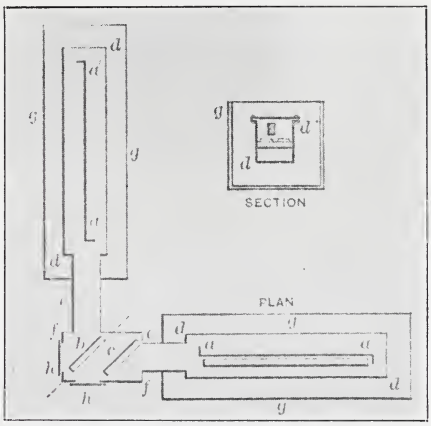


FIG. 50

Figs. 50 and 51 represent a piece of apparatus designed by Professors Morley and Rogers,[1] based on this principle. *b* and *c* (Fig. 50) are the two plane-parallel plates of the interferometer, and the two mirrors are at *a* and *a′*. Each mirror is divided into two halves as at *aa*, so that a motion of each end of the bar to be tested can be observed. The jackets *gg* serve to keep the bars at any desired temperatures. One side of the instrument, as *aa*, being kept at a constant temperature, a change in the temperature of *a′a′* will cause the fringes to move, and from this motion of the fringes the change in length, which is caused by the change in temperature, can be very accurately determined. Fig. 51 shows a perspective view of the apparatus.

Evidently the same kind of instrument is suitable for experiments in elasticity, and one of these was shown in the last lecture, where a steel axle was twisted (*cf.* Figs. 36 and

[1] MORLEY AND ROGERS, *Physical Review*, Vol. IV (1896), pp. 1, 106.

37, p. 39). If we measure the couple producing the twist, and the number of fringes which pass by, we can find the corresponding angle of twist, and a simple calculation gives us the measure of our coefficient of rigidity.

The interferometer in this second form has also been applied to the balance. Fig. 52 shows such an arrangement. The mirrors of the interferometer are on the upright metal plate, the two movable mirrors being fastened to the ends of the arms of a balance which is just visible within the horizontal box. The object of this particular experiment was to determine the constant of gravitation; in other words, to find the amount of attraction which a sphere of lead exerted on a small sphere hung on an arm of the balance. The amount of this attraction, when the two spheres are as close together as possible, is proportional to the diameter of the large sphere, which was something like eight inches. The attraction on the small ball on the end of the balance was thus the same fraction of its weight as the diameter of the large ball was of the diameter of the earth, *i. e.*, something like one twenty-millionth.[1] So the force to be measured was one twenty-millionth of the weight

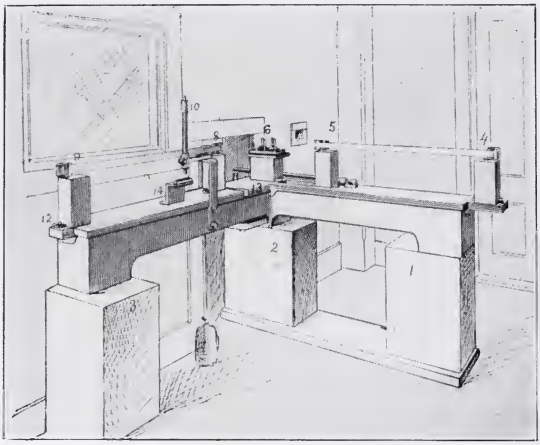


FIG. 51

[1] This ratio takes into account the increased attraction due to the greater density of the lead sphere.

of this small ball. This force is so exceedingly small that it is difficult to measure it by an ordinary balance, even if the microscope is employed. But by the interference method the approach of the large ball to the small one produced a displacement of seven whole fringes. The number of fringes can be determined to something of the order of one-twentieth of the width of one fringe. We therefore have with this instrument the means of measuring the gravitation constant, and thence the mass of the whole earth, to within about $\frac{1}{140}$ of the whole. By still more sensitive adjustment it would be possible to exceed this degree of accuracy.

FIG. 52

An instrument in which the interferometer is used for testing the accuracy of a screw is shown in Fig. 53. The screw which was to be tested by this device was intended to be used in a ruling engine for the manufacture of diffraction gratings. Now, it is necessary, in ruling gratings, to make the distance between the lines the same to within a small fraction of a micron. The error in the position of any of the lines must be less than a ten-millionth part of an inch. Ordinarily a screw from the best machinists has errors a thousand times as great. The screw must then be tested and corrected. The testing is often done with the microscope, but here the microscope is replaced by the inter-

ferometer, with a corresponding increase in the delicacy of the test.

I will conclude by showing how to measure the length of light waves by means of the interferometer. By turning

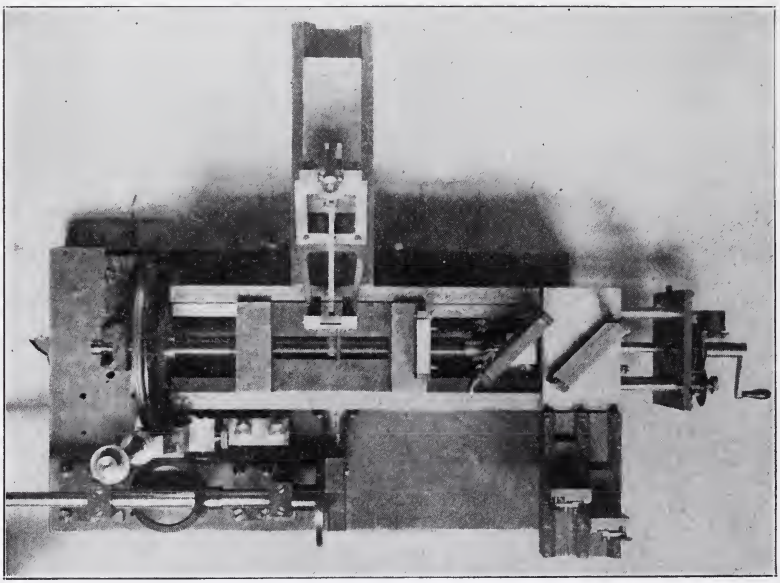


FIG. 53

the head attached to the screw, one of the interferometer mirrors (namely *C*, Fig. 39) can be moved very slowly. This motion will produce a corresponding displacement of the interference fringes. Count the number of interference fringes which pass a fixed point while the mirror moves a given distance. Then divide double the distance by the number of fringes which have passed, and we have the length of the wave. Using a scale marked from 0 to 10, made of such a size and placed at such a distance that, when a beam of light reflected from a mirror attached to the screw moves over one division, a difference in path of one-

thousandth of a millimeter has been introduced, and projecting the interference fringes upon the screen, it will be noted that while ten or twelve of these fringes move past the fiducial line the spot of light will move over a corresponding distance on the scale. In moving through ten fringes the spot of light moves through six of the divisions, and therefore the length of one wave would be six-tenths of a micron, which is very nearly the wave length of yellow light. If the light passes through a piece of red glass, and the experiment is repeated, the wave length will be greater; it is nearly sixty-seven hundredths. It is easy to see how the process may be extended so as to obtain very accurate measurements of the length of the light wave.

SUMMARY

1. A comparison between the corpuscular and the undulatory theories of light shows that the speed of light in a medium like water must be greater than in air according to the former, and less according to the latter. In spite of the inconceivable swiftness with which light is propagated, it has been possible to prove experimentally that the speed is less in water than in air, and thus the corpuscular theory is proved erroneous.

2. A number of applications of the interferometer are considered, namely, (*a*) the measurement of the index of refraction; (*b*) the coefficient of expansion; (*c*) the coefficient of elasticity; (*d*) the thickness of the "black spot;" (*e*) the application to the balance; (*f*) the testing of precision screws; (*g*) the measurement of the length of light waves.

LECTURE IV

THE APPLICATION OF INTERFERENCE METHODS TO SPECTROSCOPY

DOUBTLESS most of us, at some time or other, have looked through an old-fashioned prismatic chandelier pendant and observed that when held horizontally it produces the very curious effect of making objects appear to slope downward as though going down hill; and certainly you have all noticed the colored border which such a pendant produces at the edge of luminous objects. This experiment was made first under proper conditions by Newton, who allowed a small beam of sunlight to pass through a narrow aperture into a dark room and then through a glass prism. He observed that the sun's image was drawn out into what we call a spectrum, *i. e.*, into a band of colors which succeed one another in the well-known sequence — red, orange, yellow, green, blue, violet; the red being least refracted and the violet most.

If Newton had made his aperture sufficiently narrow and, in addition, had introduced a lens in such a way that a distinct image of the slit through which the sunlight passed was formed on the opposite wall, he would have found that the spectrum of the sun was crossed by a number of very fine lines at right angles to the direction in which the colors extended. These lines, called after the discoverer Fraunhofer's lines, have this very important characteristic, that they always appear at certain definite positions in the spectrum; and hence they were used for a considerable time for describing the location of the different colors of the spectrum. We shall endeavor roughly to present this

experiment. Not having sunlight, however, we shall take an electric arc and produce a spectrum. It will be noticed that this spectrum is not crossed by black lines, but that it is, at least for our purpose, practically continuous, as shown on Plate III, No. 1. Instead of using the electric light, let us try a source which emits but a single color. For this purpose we shall introduce into the electric arc a piece of sodium glass. Instead of a spectrum of many colors, we have one consisting mainly of one color, namely, of one yellow band. This yellow band in reality consists of two images of the slit, which are very close together, as can be shown by making the slit narrower, for then the two lines will also become narrower in proportion. If, instead of sodium glass, we introduce a rod of zinc, then, instead of one bright yellow line, the spectrum consists of lines in the red, green, and violet — two or three in the violet, one in the green, and one in the red. If we were to introduce copper, the spectrum would consist of quite a number of lines in the green; and if other substances were used, other lines would appear in the spectrum (*cf.* Plate III, Nos. 3 and 4).

Now, the lines produced by any one substance are found to occur always at a particular place in the spectrum, and are thus characteristic of the substance which produces them. If, instead of the electric light, we had used sunlight, we should find, as Fraunhofer did, that the spectrum of the sun is crossed by a number of fine, dark lines, perhaps as many as one hundred thousand, distributed throughout the spectrum. Some of the more important of these lines are shown in Fig. 54. The red end of the spectrum is at the bottom. Only the visible portion of the spectrum of the sun is shown in the figure. The pair of dark lines marked *D* coincide in position with the bright lines which are produced by sodium, as shown on Plate III, Nos. 2 and 3, and is an indication of the presence of sodium in the sun's atmosphere.

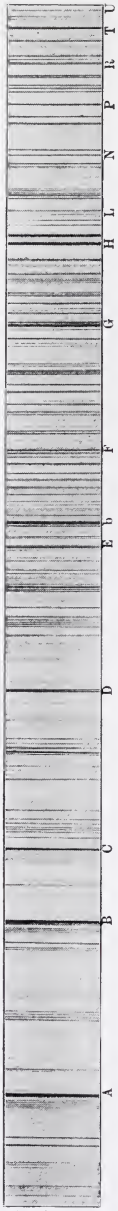


FIG. 54

As was remarked above, this sodium line is double, *i. e.*, is really made up of two lines close together. The distance between these two lines is a convenient standard of measurement for our subsequent work. This distance is so small that a single prism scarcely shows that the line is double. As we increase the number of prisms, the lines are separated more and more widely. If, instead of a prism, we use one of the best grating spectroscopes, the two lines are separated so far that we might count sixty or eighty lines between; and this fact gives a fair idea of the resolving power of these instruments. If we have two lines so close together as to be separated by only one-hundredth of the distance between these two sodium lines, the best spectroscope will hardly be able to separate them; *i. e.*, its limit of resolution has been reached.

The difference in the character of the lines from different substances is illustrated in Fig. 55. The spectrum that you have just seen is a photograph from a drawing, not a photograph from a spectrum. These are from spectra. On the right is a portion of the spectrum of iron, the other the corresponding portion of that of zinc. The enormous diversity in the appearance of the lines will be noted. Some are exceedingly fine—so fine that they are not visible at all; others are so broad that they cover ten or twenty times the distance between two sodium lines. This width of the lines depends somewhat upon the conditions under which the different substances are burned. If the incandescent vapor which sends out the lines is very dense, then the lines are very broad; if it is very

rare, then the lines are exceedingly narrow. Some of the lines are double, some triple, and some are very complex in their character; and it is this complexity of character or structure to which I wish particularly to draw your attention.

This complexity of the character of the lines indicates a corresponding complexity in the molecules whose vibrations cause the light which produces these lines; hence the very considerable interest in studying the structure of the lines themselves. In very many cases — indeed, I may say, in most cases — this structure is so fine that even with the most powerful spectroscope it is impossible to see it all. If this order of complexity, or order of fineness, or closeness of the component lines is something like one-hundredth of the distance we have adopted as our standard, it is practically just beyond the range of the best spectroscopes. It therefore becomes interesting to attempt to discover the structure by means of interference methods.

In order to understand how interference can be made use of, let us consider the nature of the interference phenomena which would be produced by an absolutely homogeneous train of waves, *i. e.*, one which consisted of only one definite simple harmonic vibration. If such a train of waves were sent into an interferometer, it would produce a definite set of fringes, and if the mirror C (Fig. 39) of the interferometer were moved so as to

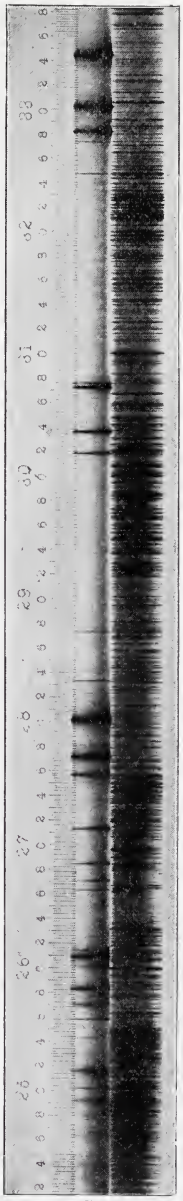


FIG. 55

increase the difference in path between the two interfering beams, then, as was explained above on p. 58, these interference fringes would move across the field of view. Now, in this case, since the light which we are using consists of waves of a single period only, there will be but one set of fringes formed, and consequently the difference of path between the two interfering beams can be increased indefinitely without destroying the ability of the beams to produce interference. It is perhaps needless to say that this ideal case of homogeneous waves is never practically realized in nature.

What will be the effect on the interference phenomena if our source of light sends out two homogeneous trains of waves of slightly different periods? It is evident that each train will independently produce its own set of interference fringes. These two sets of fringes will coincide with each other when the difference in the lengths of the two optical paths in the interferometer is zero. When, however, this difference in path is increased, the two sets of fringes move across the field of view with different velocities, because they are due to waves of different periods. Hence, one set must sooner or later overtake the other by one-half a fringe, *i. e.*, the two systems must come to overlap in such a way that a bright band of one coincides with a dark band of the other. When this occurs the interference fringes disappear. It is further evident that the difference of path which must be introduced to bring about this result depends entirely on the difference in the periods of the two trains of waves, *i. e.*, on the difference in the wave lengths, and that this disappearance of the fringes takes place when the difference of path contains half a wave more of the shorter waves than of the longer. Hence we see that it is possible to determine the difference in the lengths of two waves by observing the distance through which the mirror C must be moved in

passing from one position in which the fringes disappear to the next.

If the two homogeneous trains of waves have the same intensity, then the two sets of fringes will be of the same brightness, and when the bright fringe of one falls on the dark fringe of the other, the fringes disappear entirely. If, however, the two trains have different intensities, one set of fringes will be brighter than the other, and the fringes will not entirely disappear when one set has gained half a fringe on the other. In this case the fringes will merely pass through a minimum of distinctness. We see then that, if our source of light is double, *i. e.*, sends out light of two different wave lengths, we should expect to see the clearness or visibility of the fringes vary as the difference of path between the two interfering beams was increased.

If we invert this process and observe the interference fringes as the difference in path is increased, and find this variation in the clearness or visibility of the fringes, it is proved with absolute certainty that we are dealing with a double line. This is found to be the case with sodium light, and, therefore, by measuring the distance between the positions of the mirror at which the fringes disappear, we find that we actually can determine accurately the difference between the wave lengths of the two sodium lines. In order to carry the analysis a step farther, suppose that we magnify one of these two sodium lines. It would probably appear somewhat like a broad, hazy band. For the sake of simplicity, however, we will suppose that it looks like a broad ribbon of light with sharp edges. The distance between these edges, *i. e.*, the width of this one line, if the sodium vapor in the flame is not too dense, is something like one-fiftieth, or, perhaps, in some cases as small as one-hundredth, of the unit we have adopted — the distance between the sodium lines.

This is proved by noting the greatest difference in path which can be introduced before the fringes disappear entirely. This distance is different for different substances, and the greater it is the narrower the line, *i. e.*, the more nearly does it approach the ideal case of a source which emits waves of one period only. Now, experiment shows that the fringes formed by one sodium line will overtake those formed by the other in a distance of about five hundred waves, corresponding to about one-third of a millimeter, and that we can observe interference fringes with sodium light, under proper conditions, until the difference in path between the two interfering beams is approximately thirty millimeters. This means that the width of the band is something like one-hundredth of the distance between the two bands. The width of a single line can be appreciated in the ordinary spectroscope when the sodium vapor is dense, and under these conditions the fringes vanish when the difference in path is only one-half inch, or even less. When we try to make the source bright by increasing the temperature and density of the sodium vapor in the flame, the band broadens out to such an extent that the difference in path over which interference can be observed may be less than one-hundredth of an inch.

The above discussion of the case of the two sodium lines may easily be extended to include lines of greater complexity, and it will be found that, whatever the nature of the source, the clearness or visibility of the fringes will vary as the difference in path between the two interfering beams is increased. It may also be shown that each particular complex source will show variations in the visibility of the fringes which are peculiar to it.

Inversely it is evident that by the observation of the character of the curve which expresses the relation between the clearness of the fringes and the difference of path—the

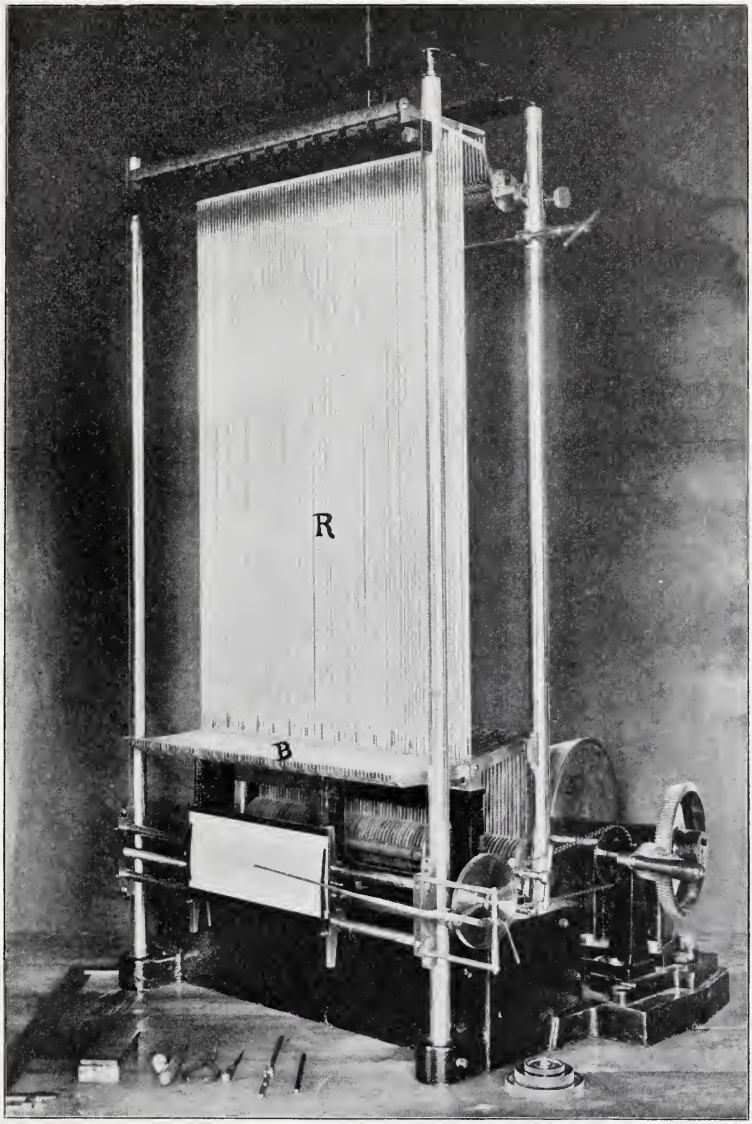


FIG. 56

visibility curve, as it may be termed — we can draw conclusions as to the character of the radiations which cause the interference phenomena, even when such investigation is beyond the power of the best spectroscopes. In order to make the method (it may perhaps be called the method of light-wave analysis) an accurate process, it is necessary, in the first place, to produce a number of visibility curves from known sources. Thus, for example, we may take two lines corresponding to the sodium lines, and produce their visibility curve, as we did before, by adding up the separate fringes and obtaining the resultant; we may then take three or four or any number of lines, and determine the corresponding visibility curves. Each of these, instead of being a single line, may have an appreciable breadth, and the brightness of the line may be distributed in various ways within the breadth.

Now, the process of adding up such a series of simple harmonic curves (for the interference fringes are represented by simple harmonic curves) is very laborious. Hence the instrument shown in Fig. 56, called a *harmonic analyzer*, was devised to perform this work mechanically. It looks very complex; in reality it is very simple, the apparent complexity arising from the considerable number of elements required. A single element is shown in Fig. 57. A curved lever which is pivoted at o is represented at B. One end of this lever is attached to the collar of the eccentric A. When this eccentric revolves, it therefore transmits to the lever B a motion which is very nearly simple harmonic. The amount of the motion which is communicated to the writing lever u is regulated by the distance of the connecting rod R from the axis o. When the connecting rod is on one side of the axis the motion is positive; when on the other side the motion would be negative. The end of this lever is connected to another lever x, and the farther

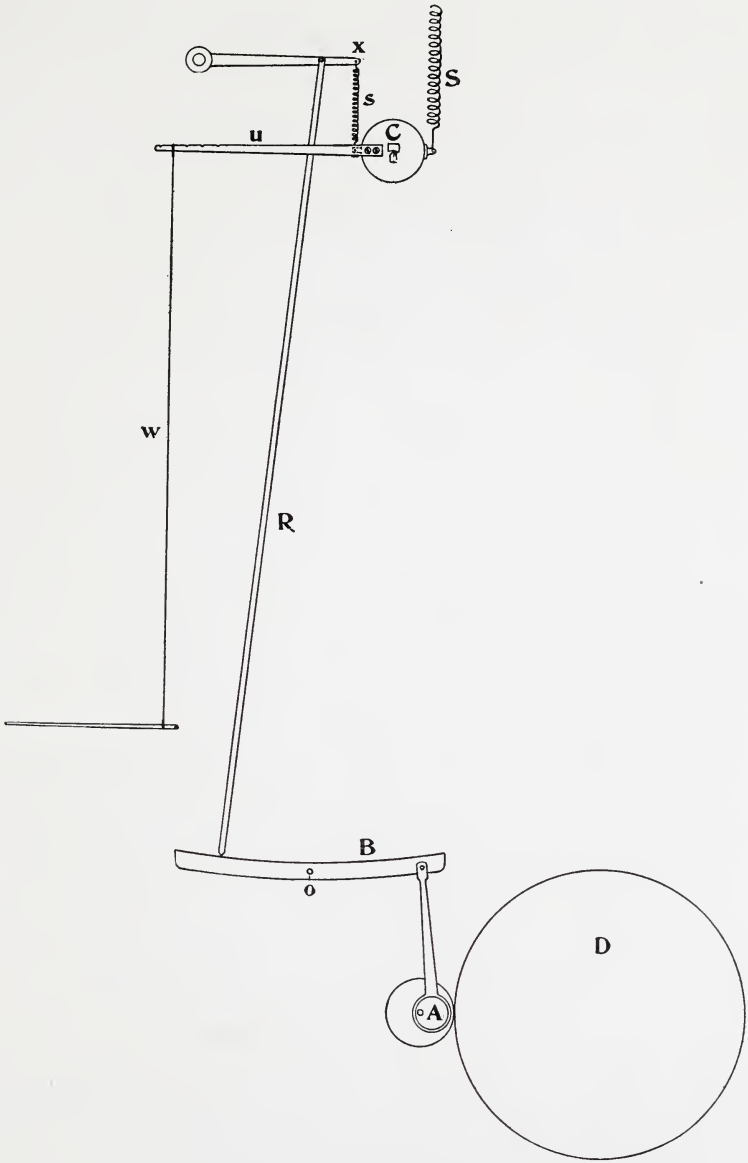


FIG. 57

end of this lever is connected with a small helical spring *s*. There are eighty such elements arranged in a row, as shown in Fig. 56. In order to add the force of all of the springs, they are connected with the drum *C*, which can turn about its axis, and counterbalanced by a very much larger spring *S* connected to the other side of the drum. This gives us the means of adding forces which are proportional to the amount of displacement of the lever below, and hence the sum of the forces of these eighty springs is in direct proportion (at any rate to a close degree of approximation) to the sum of the motions themselves. We have thus a mechanical device for adding simple harmonic motions.

To illustrate this addition of simple harmonic motions by means of our machine, one of the connecting rods is first moved out to the extreme end of the lever. We shall then have but one simple harmonic motion to deal with, and this corresponds to an absolutely homogeneous source. The resulting curve is the first one in Fig. 58. Each one of the oscillations corresponds to an interference fringe, and there would be an infinite number of such if the difference in path were indefinitely increased. Now we will take the case of two simple harmonic motions. At *b*, curve 2, the fringes have disappeared completely. One series of fringes has just overtaken the other by one-half a fringe, and, therefore, they neutralize each other. At *c* the fringes have begun to appear again, and at *d* they have attained a maximum visibility or clearness. They then disappear and reappear again, and so on indefinitely.

Curve 3 represents the case of the two sodium lines, each of which is supposed to be double. It will be observed that in this case there are two periods; one, the same as that of curve 2, which corresponds to the double sodium line, and the other a longer period whose first minimum occurs at *e* and which corresponds to the shorter distance between the

two components of each line. The conclusion which can be drawn from observation of such a curve as this is that the

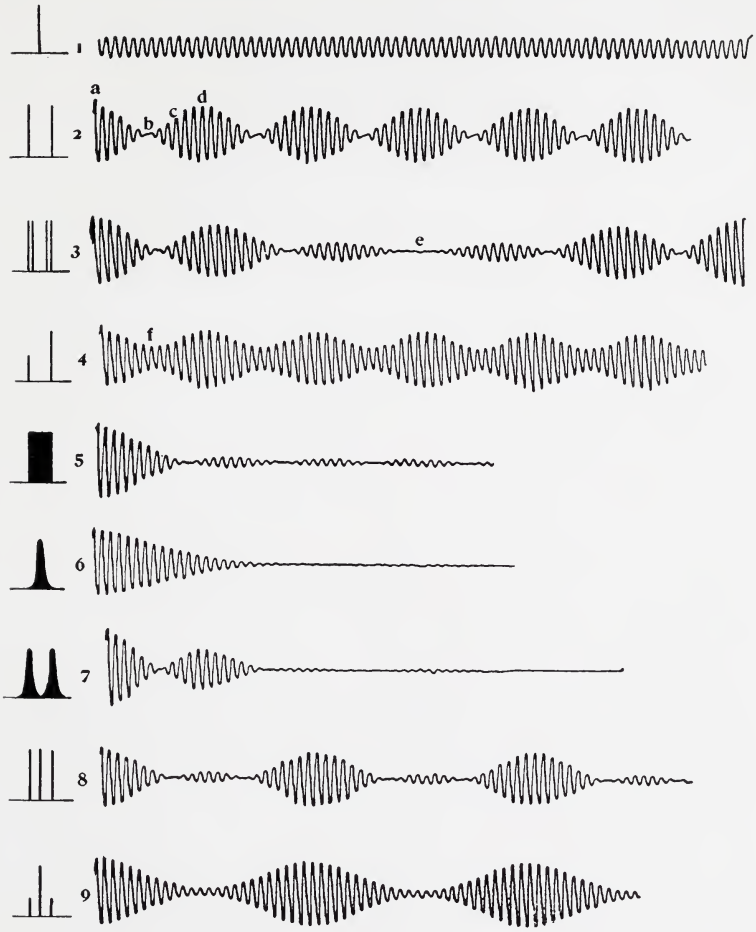


FIG. 58

source which was used in obtaining it was a double line, each of whose components was double.

Curve 4 represents the visibility curve of two lines, one of which is very much brighter than the other, but whose

distance apart is the same as that of the lines of curve 2. The period of the visibility curve is the same as that of 2, but instead of going to zero it merely goes to a minimum at *f*. Inversely, when we get such a curve as this we know that one of the lines is brighter than the other—just how much brighter can be learned from the ratio of the maximum and minimum ordinates.

Curve 5 is that due to a single broad source of uniform intensity throughout. It will be noted how quickly the fringes lose their distinctness. Curve 6 is that due to a broad source which is brighter in the middle than at the edges. The distribution in this case is supposed to follow the exponential law. The corresponding visibility curve does not exhibit maxima and minima, but gradually dies out and remains at zero. Curve 7 corresponds to a double source each of whose components is brighter in the middle. Curve 8 represents a triple source each of whose components is a simple harmonic train of waves of the same intensity. Curve 9 represents the visibility due to a triple source in which the outer components are much fainter than the middle one.

We might go on indefinitely constructing on the machine the visibility curves which correspond to any assumed distribution of the light in the source. The curves presented will suffice to make clear the fact that there is a close connection between the distribution of light in any source and the visibility curve which can be obtained with the use of that source. It is, however, the inverse problem, *i. e.*, that of determining the nature of the source from observation of the visibility curve, in which the greatest interest lies.

In order to determine by this method the character of the source with which we are dealing, we must find our visibility curve by turning the micrometer screw of the inter-

ferometer and noting the clearness of the fringes as the difference of the path varies. We then construct a curve which shall represent this variation of visibility on a more or less arbitrary scale, and compare it with one of the known forms, such as those shown in Fig. 58. There is, however, a more direct process. The explanation of this process involves so much mathematics that I shall not undertake it here. It will be sufficient to state that the harmonic analyzer cannot only be used as has been described, but is also capable of analyzing such visibility curves. Thus, if we introduce into the instrument the curve corresponding to the visibility curve, by making the distances of the connecting rods from the axis proportional to the ordinates of the visibility curve, and then turn the machine, it produces directly a very close approximation to the character of the source. For example, take curve 2 of Fig. 58. By its derivation we know that it corresponds to a double source each of whose components is absolutely homogeneous. If we introduce this curve, or rather the envelope of it, into the machine, it will give a resultant which represents the character of the source to a close degree of approximation. The actual result is shown in Fig. 59, in which the ordinates represent the intensity of the light. We thus see that the machine can operate in both ways, *i. e.*, that it can add up a series of simple harmonic curves and give the resultant, which in the case before us is the visibility curve, and that it can take the resultant curve and analyze it into its components, which here represent the distribution of the light in the source.

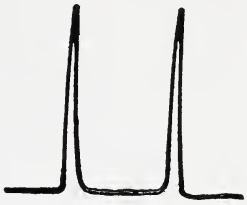


FIG. 59

Now the question naturally arises as to how the observations by which the visibility curve is determined are conducted; also as to what units to adopt, and what scale

of measurement. It is apparently something very indefinite. The visibility is not a quantity that can be measured, as we can a distance or an angle — unless, to be sure, we first define it. After defining it properly, we can produce, in accordance with that definition, interference fringes that shall have any desired visibility. By the use of fringes which have a known visibility we can educate the eye in estimating visibility, or we may have these standard fringes before us for comparison at the time of observation, and may then determine when the two systems are of the same clearness; and when they are of the same clearness, we say that the desired visibility is the same as that whose value is known from our formula. This is the more accurate method, and is the one which was finally adopted; but long before its adoption it was found that fairly accurate visibility curves could be obtained by merely agreeing to call the visibility 100 when it was perfect, 75 when good, and 50 when fair. Then 25 would be rather poor, 10 would be bad, and at zero the fringes would vanish. Of course, there would be a greater or less difference in what we should agree to call good, but in general we can tell where the fringes were half as clear as their perfect value, provided, of course, we had this perfect value given, etc.

As a matter of fact, however, it is not of the utmost importance to determine the visibility with great accuracy. We know that we can measure a minimum or a maximum independently of any scale, and these points are the really important ones. For example, a curve may come to zero gradually or abruptly — in both cases the distance between the two lines which produced the curve would be exactly the same. The two pairs might differ in character in other ways, but the distance between the two components of each pair would be the same. So, even without an absolute scale that we have tested, and even without any very great amount

of experience in observation, we can get a very fair visibility curve, and from that a very fair conception of the nature of the spectrum of the particular source we are examining, by merely determining the points of maximum and minimum clearness.

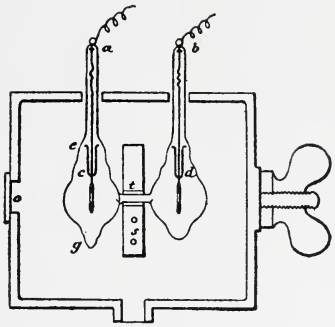


FIG. 60

Before discussing some of the visibility curves that have been obtained, I should like to say a word concerning the source of light. When the source is under ordinary conditions, *i. e.*, under atmospheric pressure, the molecules are not vibrating freely, and disturbing causes come in to make the oscillations not perfectly homogeneous. Hence the light from such a source, instead of being a definite, sharp line, is a more or less diffuse band. In order to obtain the character of the line under the extreme conditions, *i. e.*, under as small pressure as possible, the substance must be placed in a vacuum tube. The tube is then connected to an air pump and exhausted until the pressure in it is reduced to a few thousandths of an atmosphere.

When the exhaustion has become sufficient — the time depending on the particular degree of exhaustion required by the substance which we wish to examine — the tube is heated to drive off the remaining water vapor, sealed up, and is then ready for use. The residual gas is made luminous by the spark from an induction coil. In some cases the substance is sufficiently volatile to show the spectrum at ordinary temperatures; *e. g.*, that of mercury appears after slight heating. In the case of such substances as cadmium and zinc the tube is placed in a brass box, as illustrated in Fig. 60, and heated until the substance is volatilized, a thermometer giving us an idea of the temperature reached.

Fig. 61 illustrates the arrangement of the apparatus as it is actually used. An ordinary prism spectroscope gives a preliminary analysis of the light from the source.

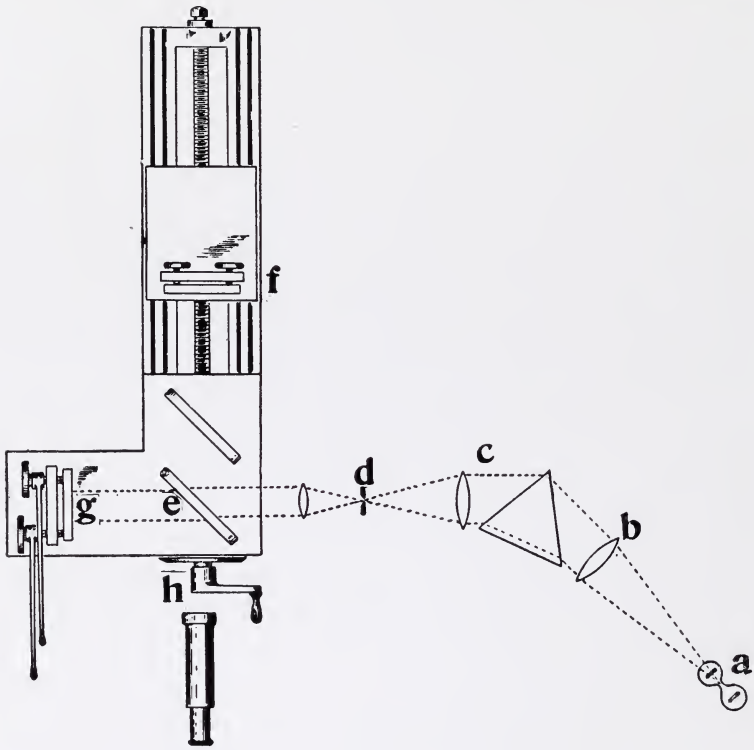


FIG. 61

This is necessary because the spectra of most substances consist of numerous lines. For example, the spectrum of mercury contains two yellow lines, a very brilliant green line, and a less brilliant violet line. If we pass all the light together into the interferometer, we have a combination of all four. It is usually better to separate the various radiations before they enter the interferometer. Accordingly, the light from the vacuum tube at *a* passes through an

ordinary spectroscope *bcd*, and the light from only one of the lines in the spectrum thus formed is allowed to pass through the slit *d* into the interferometer.

As explained above, the light divides at the plate *e*, part going to the mirror *f*, which is movable, and part passing through to the mirror *g*. The first ray returns on the path *feh*. The second returns to *e*, is reflected, and passes into the telescope *h*. If the two paths are exactly equal, we have interference phenomena in white light; but for monochromatic light the difference of path (from the point *e* to the mirror *f*, and from the same point to the mirror *g*) may be very considerable. Indeed, in some cases interference can be obtained when the difference in the two paths amounts to over half a million waves.

It is rather important to note that the surface of the mirror *g* must be so set by means of the adjusting screws at its back that its image in the mirror *e* shall be parallel with the surface of the movable mirror *f*. When this is the case the fringes, instead of being straight lines, as in the case of the fringes in white light, are concentric circles very similar in appearance to Newton's rings. Having thus adjusted the interferometer so that the fringes are circles, the difference in path is increased by turning the micrometer screw a definite amount, say half a millimeter at a time. At every half millimeter an observation is taken of the visibility, and then these readings are plotted on co-ordinate paper as ordinates, the corresponding difference of path serving as abscissæ. The ends of these ordinates trace out the visibility curve. This curve is then set on the harmonic analyzer, as described above, and the machine turns out the curve corresponding to the distribution of the light in the line examined.

In this way the radiations of many substances were analyzed, and in almost every case it was found that the

line was not produced by homogeneous vibrations, but was double, treble, or even more complex. The distances between the components of these compound lines are so small that it is practically impossible, except in a few cases, to observe them in the ordinary spectroscope.

The following diagrams (Figs. 62–8) present a number of these visibility curves. Thus Fig. 62 represents that obtained from the red radiation of hydrogen. The curve to the right represents the visibility curve, while on the left the corresponding distribution of the light is drawn. Beginning at a difference of path zero, the visibility was

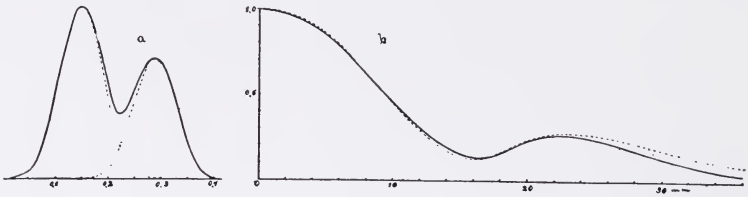


FIG. 62

100, and at one millimeter it was somewhat less, and so on, until at about seventeen millimeters we find a minimum. As the difference in path increases, we find that there is a maximum at twenty-three millimeters. After that the curve slopes down, and at about thirty-five millimeters it disappears entirely. Since the curve is periodic, we may be pretty sure that this red line of hydrogen is a double line. This fact, I believe, has never yet been observed, though the distance between the two components is not beyond the range of a good spectroscope, being about one-fortieth or one-fiftieth of the distance between sodium lines.[1]

Fig. 63 represents the curve which was obtained from sodium vapor in a vacuum tube. When we burn sodium at atmospheric pressure—as, for example, when we place sodium

[1] This prediction has since been amply confirmed by direct observation.

glass in a Bunsen flame—the visibility curve due to its radiations diminishes so rapidly that it reaches zero when the difference of path is about forty millimeters; it is practically

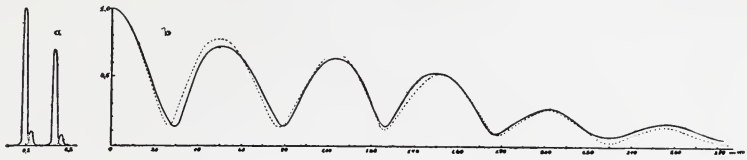


FIG. 63

impossible to go farther than this. It is seen that the curve is periodic, which would indicate that each one of the sodium lines is a double line. The intensity curve at the left represents one of the sodium lines only. The other, on the same scale, would be distant about half a meter. We can from this get some idea of the relative sensitiveness of this process of light-wave analysis, as compared with that of ordinary spectrum analysis. It will be observed that the intensity curve shows still another small component which corresponds to still another longer period, but the existence of these short companion lines is not absolutely certain.

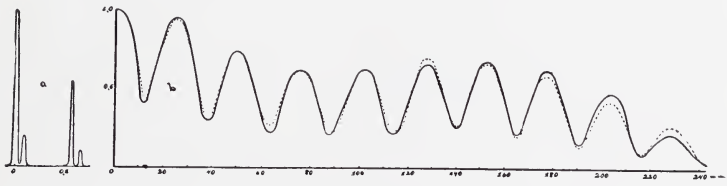


FIG. 64

Fig. 64 represents the curve of thallium. The oscillation shows that it is a double line, and not very close. The distance between the components is about one-sixtieth of the distance between the sodium lines. We have also a longer oscillation which shows that each one of the components is

double. The distance between these small components and the larger ones is something like one-thousandth of the distance between sodium lines, corresponding to a separation of lines far beyond the possible limit of the most powerful spectroscope.

The curve of the green radiation of mercury is shown in Fig. 65. This curve is really so complicated that the character of the source is still a little in doubt. The machine has not quite enough elements to resolve it satisfactorily, having but eighty when it ought to have eight hundred. The curve

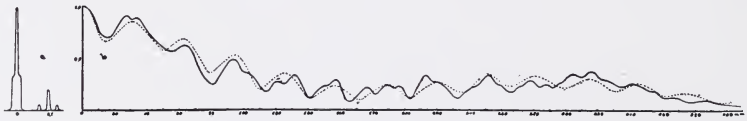


FIG. 65

looks almost as though it were the exceptional result of this particular series of measurements, and we might imagine that another series of measurements would give quite a different curve. But I have actually made over one hundred such measurements, and each time obtained practically the same results, even to the minutest details of secondary waves. The nearest interpretation I can make as to the character of the spectral source is given at the left of this diagram. It will be noticed that the width of the whole structure is, roughly speaking, one-sixtieth of the distance between the sodium lines. The distance between the close components of the brighter line is of the order of one-thousandth of the distance between the sodium lines. The fringes in this case remain visible up to a difference of path of 400 millimeters, and they have actually been observed up to 480 millimeters, or nearly one-half meter's difference in path — corresponding to something like 780,000 waves.

In the curve of Fig. 66 we have quite a contrast to the preceding. Here we have a radiation almost ideally homogeneous. Instead of having numerous maxima and minima like the curves we have been considering, this visibility curve diminishes very gradually according to a very simple mathematical law, which tells us that the source of light is a single line of extremely small breadth, the breadth being of the order of one eight-hundredth to one-thousandth of the

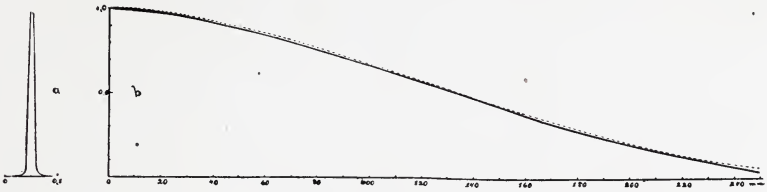


FIG. 66

distance between the sodium lines. It is impossible to indicate exactly the width of the line, because the distribution of intensity throughout it is not uniform. The important point to which I wish to call attention, however, is that this curve is of such a simple character that for a difference of path of over 200 millimeters, or 400,000 light waves, we can obtain interference fringes. This indicates that the waves from this source are almost perfectly homogeneous. It is therefore possible to use these light waves as a standard of length, as will be shown in a subsequent lecture. The curve corresponds to the red radiation from cadmium vapor in a vacuum tube. In using this red cadmium wave as a standard of length it is very important to have other radiations by which we can check our observations. The cadmium has two other lines, which serve as a control or check to the result obtained by the first.

Fig. 67 represents the green radiation of cadmium. This curve is not quite so simple as that of the red, but

extends almost to 200 millimeters. The corresponding line is shown to be a close double.

The curve corresponding to the violet light of cadmium is shown in Fig. 68, and is seen to be comparatively simple.

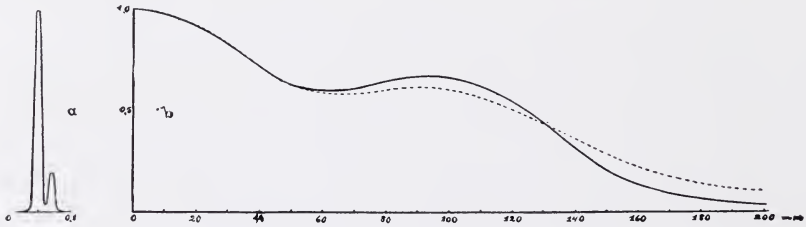


FIG. 67

We have thus shown that spectral lines are complex distributions of light, whose resolution in general is beyond the power of the spectroscope. This complexity of the spectral lines is particularly interesting because it indicates a corresponding complexity of the molecules which cause the vibrations which give rise to the corresponding spectral lines.

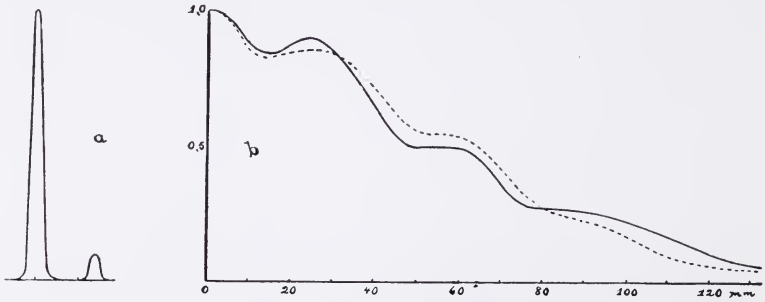


FIG. 68

This complexity may be likened to the complexity of a solar system; and while this may bring dismay to the Keplers and Newtons who may hope to unravel the mysteries of this pigmy world, it certainly increases the interest in the problem.

SUMMARY

1. The spectrum of the light emitted by incandescent gases is not continuous, but is made up of a number of bright lines whose position in the spectrum is very definite, and which are characteristic of the elements which produce them.

2. These "lines" are not such in a mathematical sense, but have an appreciable width and a varying distribution of light, and in some cases are highly complex.

3. This variation in distribution is, however, restricted to such narrow limits that in most cases it is impossible to investigate it by the best spectroscopes; but by the method of visibility curves the lines may be resolved into their elements.

4. An important auxiliary for the interpretation of the visibility curves is the harmonic analyzer—an instrument which sums up any number of simple harmonic motions, and which also analyzes any complex motion into its simple harmonic elements.

LECTURE V

LIGHT WAVES AS STANDARDS OF LENGTH

In the last lecture it was shown that in many cases the interference fringes could be observed with a very large difference in path—a difference amounting to over 500,000 waves. It may be inferred from this that the wave length, during the transmission of 500,000 or more waves, has remained constant to this degree of accuracy; that is, the waves must be alike to within one part in 500,000. The idea at once suggests itself to use this invariable wave length as a standard of length. The proposition to make use of a light wave for this purpose is, I believe, due to Dr. Gould, who mentioned it some twenty-five years ago. The method proposed by him was to measure the angle of diffraction for some particular radiation—sodium light, for example—with a diffraction grating. If we suppose Fig. 69 to represent, on an enormously magnified scale, the trace of such a grating, then the light for a particular wave length—say one of the sodium lines—which passes through one of the openings in a certain direction, as AB, is retarded, over that which passes through the next adjacent opening, by a constant difference of path; and therefore in the direction AB all the waves, even those which pass through the last of a very large number of such apertures, are in exactly the same phase. There will be then, if we are observing in a spectrum of the first order, as many waves in this distance AB as there are apertures in the distance AC. A diffraction grating is made by ruling upon a glass or a metal surface a great number of very fine lines by a ruling diamond, the number being recorded by the ruling-machine

itself, so that there can be no error in determining the number of rulings. This number is usually very large, between 50,000 and 100,000. Since this number of lines is accurately known, we know also the number of spaces in the whole distance AC. This distance can be measured by comparing the two end rulings with an intermediate standard of length, which has been compared with the standard yard or the standard meter with as high a degree of accuracy as is possible in mechanical measurements. If, now, we know also the angle ACB, we can calculate the distance AB; and since we know the number of waves in this distance, which is the same as the number of apertures, we have the means of measuring the length of one wave. It will be observed, in making such an absolute determination of wave length by this means, that we have to depend entirely upon the accuracy of the distance between the lines on the grating—a distance which is measured by a screw advancing through a small proportion of its circumference for each line ruled. If the intervals between the lines are not exactly equal, then there will be an error introduced, notwithstanding every precaution taken, which it is extremely difficult, if not impossible, to correct.

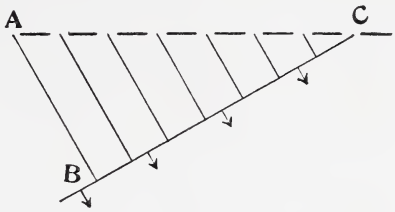


FIG. 69

Another error may be introduced in making the comparison of the two extreme lines on the grating with the standard decimeter. This error may, roughly, be said to amount to something like one-half a micron, *i. e.*, one-half of one-thousandth of a millimeter. If, then, the entire length of the ruling is fifty millimeters, and the error, say, one ten-thousandth of a millimeter, the wave length may be measured to within one part in 500,000. This is the error

upon the supposition that our standard is absolutely correct. But the length of the standard decimeter itself has to be determined by means of microscopic measurements, and since the temperature plays a considerable rôle, it is difficult to avoid errors very much larger than those due to the microscope. If we combine all these errors, we can probably attain at best an accuracy in all measurements involved of the order of one part in 100,000. Finally, we have to measure the angle ACB, and it is very much more difficult to measure angles than lengths. All these errors—the measurement of the angle, the error in the determination of the distance AC, that in the comparison of the intermediate standard which we use, and that in the distribution of these spaces—may combine in such a way that the total error may amount to very much more than one part in 100,000; it may be one in 20,000 or 30,000. This degree of accuracy, however, is greater than that attained by either of the other two methods which have been proposed for establishing an absolute standard of length.

The first of these proposed standards was the length of the pendulum which vibrates seconds at Paris. Such a pendulum may be obtained by suspending from a knife edge a steel rod upon which a large lens-shaped brass bob is fastened. The steel rod carries another knife edge near the other end, so that the pendulum can be turned over so as to be suspended from this lower knife edge. The pendulum must then be adjusted so that its time of vibration is exactly the same in either position, which can be done with but little difficulty. When such a pendulum vibrates seconds in either position, the distance between the knife edges is the length of a simple seconds pendulum.

We may also construct a simple pendulum by fastening a sphere of metal to the end of a thin, fine wire. It is then necessary to measure the time of oscillation, and the distance

between the point of suspension and the center of gravity of the spherical bob. This distance can be measured to a very fair degree of accuracy. Unfortunately the different observations vary among themselves by quantities even greater than the errors of the diffraction method.

The second of these proposed standards was the circumference of the earth measured along a meridian, as it was believed that this distance is probably invariable. There are, however, certain variations in the circumference of the earth, for we know that the earth must be gradually cooling and contracting. The change thus produced is probably exceedingly small, so that the errors in measuring this circumference would not be due so much to this cause as to others inherent to the method of measuring the distance itself. For suppose we determine the latitude of two places, one 45° north of the equator and one 45° south. The difference in latitude of these places can be determined with astronomical precision. The distance between the places is one-fourth of the entire circumference of the earth. This distance must be measured by a system of triangulation — a process which is enormously expensive and requires considerable time and labor; and it is found that the results of these measurements vary among themselves by a quantity even greater than do those reached with the pendulum. So that none of these three methods is capable of furnishing an absolute standard of length.

While it was intended that one meter should be the one forty-millionth of the earth's circumference, in consequence of these variations it was decided that the standard meter should be defined as the arbitrary distance between two lines ruled on a metal bar a little over a meter long, made of an alloy of platinum and irridium. It was made of these two substances principally on account of hardness and durability. In order to bring the metal as nearly as possible

to what was termed its "permanent condition," these bars were subjected to all sorts of treatment and maltreatment. The originals were cast and recast a great many times, and afterward they were cooled — a process which took several months.

After such treatment it is believed that the alteration in length of these bars will be exceedingly small, if anything at all. But, as a matter of fact, it is practically impossible to determine such small alterations, because, while there have been a number of copies made from this fundamental standard, these copies are all made of the same metal as the original; hence, if there were any change in the original, there would probably be similar changes in all the copies simultaneously, and it would therefore be impossible to detect the change. The extreme variation, however, must be of the order of one-thousandth of a millimeter or less in the whole distance of 1,000 millimeters.

The question rightly arises then: Why require any other standard, since this is known to be so accurate? The answer is that the requirements of scientific measurement are growing more and more rigorous every year. A hundred years ago a measurement made to within one-thousandth of an inch was considered rather phenomenal. Now it is one of the modern requirements in the most accurate machine work. At present a few measurements are relied upon to within one ten-thousandth of an inch. There are cases in which an accuracy of one-millionth of an inch has been attained, and it is even possible to detect differences of one five-millionth of an inch. Past experience indicates that we are merely anticipating the requirements of the not too distant future in producing means for the determination of such small quantities. Again, in order that the results of scientific work already completed, or shortly to be completed, may be compared

and checked with those of subsequent researches, it is essential that the units and standards employed should have the same meaning then as now, and, therefore, that such standards should be capable of being reproduced with the highest attainable order of accuracy. We may, perhaps, say that the limit of such attainable accuracy is the accuracy with which two of the standards can be compared, and this is, roughly speaking, about one-half of a micron—some say as small as three-tenths of a micron. For such work neither of the three methods described above of producing a standard is sufficiently accurate. As before stated, the results obtained by them vary among themselves by quantities of the order of one part in 50,000 to one part in 20,000. Since the whole meter is 1,000,000 microns, an order of accuracy of one-half of a micron, which can be obtained with a microscope, would mean one part in 2,000,000, which is far beyond the possibilities of any of the three methods proposed.

We now turn to the interference method. Some preliminary experiments showed that there were possibilities in this method. The fact to which we have just drawn attention—namely, that the wave lengths are the same to at least one part in 500,000—looks particularly promising and leads us to believe that, if we had the proper means of using the waves and of multiplying them up to moderately long distances, without multiplying the errors, they could be used as a standard of length which would meet the requirement. This requirement is that a sufficient number of waves shall produce a length which may be reproduced with such a degree of accuracy that the difference between the new standard and the one now serving as the standard cannot be detected by the microscope.

The process is, in principle, an ideally simple one, and

consists in counting the number of waves in a given distance. However, in counting such an enormous number, of the order of several hundred thousands, one is liable to make a blunder — not an error in a scientific sense, but a blunder. Of course, ultimately, this would be detected by the process of repetition.

The investigation, in a concrete form, presents a number of interesting points, involving problems of construction of a unique character which had to be solved before the process could be said to be perfectly successful.

The construction and operation of the apparatus will be much more readily understood if we first dwell a little upon the conditions that are to be fulfilled. Suppose, for illustration, that it is required to find the distance between two-mile posts on a railroad track. The most convenient method for measuring such a distance would be by a hundred-foot steel tape stretched by a known stretching force and applied to the steel rails. The rails are mentioned simply in order that there should not be any sag of the tape which would introduce still another error. The zero mark of the tape being placed against a mark on the rail which serves as the starting-point, a second mark is made on the rail opposite the hundred-foot mark of the tape. The tape is then placed in position a second time with one end on the second mark, and a third mark is placed at the farther end; and so on indefinitely. This is the first process. By it we have divided the mile into the nearest whole number of hundred-foot spaces. Then we measure the fractions.

The second operation consists in verifying the length of the steel tape, which we must do by comparing it with a standard yard or foot by the same stepping-off process.

The process of measuring the meter in light waves is essentially the same as that described above, the meter answering to the distance of a mile of track, and the

hundred-foot tape corresponding to a considerably smaller distance. This smaller distance is what I have termed an "intermediate standard." There is in this latter case the additional operation of finding the number of light waves in the intermediate standard; so that, in reality, there are three distinct processes to be considered.

In the first operation it is evident that, if an error is committed whenever we lift the tape and place it down again, the smaller the number of times we lift it and place it down, the smaller the total error produced; hence, one of the essential conditions of our apparatus would be to make this small standard as long as possible. The length of the intermediate standard is, however, limited by the distance at which we can observe interference fringes. The limit, as was stated in the last lecture, is reached when this distance is of the order of several hundred thousand waves. At this distance the interference fringes are rather faint, and it seemed better for such determinations not to make use of the extreme distance, but of such a smaller distance as would insure distinct interference fringes. It was found convenient to use, as the maximum length of the intermediate standard, one decimeter. The number of light waves in the difference of path (which is twice the actual distance, because the light is reflected back) would be something of the order of three or four hundred thousand waves. With such a difference of path we can still see interference fringes comparatively clearly, if we choose the radiating substance properly.

The investigations described in the last lecture showed that the radiations emitted by quite a number of the substances which were examined were more or less highly complex. One remarkable exception, however, was found in the red radiation of cadmium vapor. This particular radiation proved to be almost ideally homogeneous, *i. e.*, to con-

sist very nearly of a series of simple harmonic vibrations. This radiation was therefore eminently suited to the purpose, and was adopted as the standard wave length.

Most substances produce a more or less complicated spectrum involving quite a number of lines, but in the case of cadmium vapor, though there are three different radiations, these three are all so nearly homogeneous that each one can be used; and the complexity of the spectrum is in this case an advantage, as will be shown below. To produce the cadmium radiation, metallic cadmium is placed in a glass tube which contains two aluminum electrodes. The tube is then connected by glass tubing with an air pump and exhausted of air. The tube is also heated so as to drive off all residual gas and vapor, and when the required degree of exhaustion is reached, it is hermetically sealed and in condition to use. The cadmium is not very volatile, and at ordinary temperatures we should see scarcely anything of the cadmium light when the electric discharge passes. The tube is therefore placed in a metal box, as shown in Fig. 60, which is furnished with a window of mica and has a thermometer introduced into one side. If the box be heated by a Bunsen burner to a temperature in the neighborhood of 300° C., the cadmium vapor fills the tube, and can then be rendered luminous by the passage of the electric spark.

Now, it is found most convenient not to make this first intermediate standard in the form of a bar like the standard meter, with two lines drawn upon it; for then we should introduce errors of the microscope at every reading, and these errors would be added together. Thus, since this is one-tenth of the whole meter, we might have, in adding up, ten times the error of the microscope, which we said was of the order of one-half a micron; we could thus have, in the end, an error of five microns. The interference method gives us the means of multiplying the length of the intermediate

standard with the slightest possible error, amounting, perhaps, to one-twentieth of a micron; in some cases a little less. If two plane surfaces be parallel to one another and a given distance apart, then, with the interferometer, we may locate the position of either one of these surfaces by means of the interference fringes in white light to within one-twentieth of a fringe, which means one-fortieth of a wave, or one-eightieth of a micron. It has been found most convenient to use glass surfaces very carefully polished and made as nearly plane as possible, and silvered on the front. The two surfaces are mounted on a brass casting, and carefully adjusted so as to be as nearly parallel as possible, so that it does not matter what part of the surface is used. This parallelism of the two surfaces must be arranged with extraordinary accuracy; the greatest deviation from true parallelism must be of the order of one-half of a fringe, which would be one-fourth of a wave length, or one-eighth of a micron. Since the width of the surface is something like two centimeters, the allowable angle between the two surfaces is something like one part in two hundred thousand.

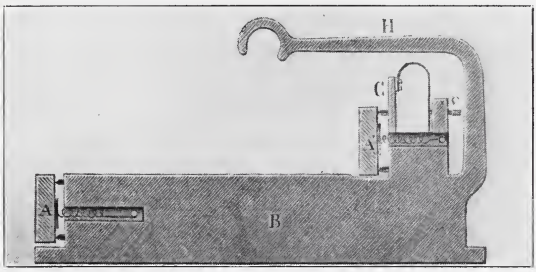


FIG. 70

A section of the intermediate standard we have been describing is represented in Fig. 70. The two glass surfaces are about two centimeters square and silvered on their front surfaces, which are very nearly true planes. Their rear surfaces press against three small pins. These are adjusted

for parallelism by filing until the requisite degree of accuracy is obtained. The parallelism cannot be made altogether perfect, and, as a matter of fact, in some cases the error may amount to as much as one-tenth of a micron or more.

Fig. 71 represents a perspective view of the same thing. In this figure the intermediate standard rests on a carriage by means of which it may be moved as necessary for the purpose of comparing it with the whole meter. In making this comparison the surfaces must be parallel to the mirror which serves as a reference plane in the interferometer. The parallelism in this case must be of the same order of accuracy as that between the surfaces themselves. The adjustment is made by the screws at the rear, one of which turns the whole standard about a vertical axis and the other about a horizontal one.

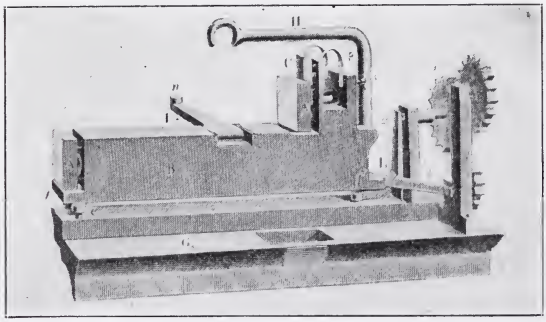


FIG. 71

In determining the number of waves in the meter, the first operation is to find the number of whole waves in this intermediate standard. It can readily be conceived that the counting of something like 300,000 waves would be no small matter; in fact, a little calculation would show that, if we counted two per second, it would take over forty hours to make the count. Probably a number of methods will suggest themselves of making such a process of counting automatic. Indeed, several experiments have been made, and with some promise of success; but the possibility

of skipping over one fringe, through some accident, is serious. It was therefore thought desirable to use another process, very much longer and more tedious, but very much surer. This process consists in dividing the distances to be measured into a very much smaller number of parts, so that the distances to be measured in waves would be very much smaller. Thus a distance of ten centimeters contains 300,000 waves; half of this distance would contain 150,000. If we go on dividing in this way, until we get to the last one of nine such steps, we reach an intermediate standard whose length is something of the order of one-half millimeter. The total number of waves in this standard is about 1,200, and this number it is a comparatively simple matter to count.

The method of proceeding in counting these fringes is the same as that described above. The reference plane, as we will call the movable mirror in the interferometer, is moved gradually from coincidence with the first surface to coincidence with the second, and the fringes which pass are counted. Such a count was made for the three standard radiations, namely, the red, green, and blue of cadmium vapor. The result was 1,212.37 for red, 1,534.79 for green, and 1,626.18 for blue. Now, an important point is that we can measure these fractions with an extraordinary degree of accuracy; so the second decimal place is probably correct to within two or three units. The whole number we know to be correct by repeating the count and getting the same result. Having thus obtained this number, including also the fractions of waves on the shorter standard to a very close approximation, we compare it with the second, which is, approximately, twice as long. This comparison gives us, without further counting, the whole number of waves in the second standard by multiplying the number in the first by two. We have the same possibility of measuring fractions on the second standard, and so can determine

the number of waves in its length with an equal degree of accuracy.

I will give the description of this process somewhat more in detail. In Fig. 72 mm' represents the first or the shorter standard viewed from above. This standard rests on a carriage which can be moved with a screw. The second standard nn' is twice as long as the first, and is placed as close as possible to the first and rigidly connected with some part of the frame. The mirror d is the reference plane.[1]

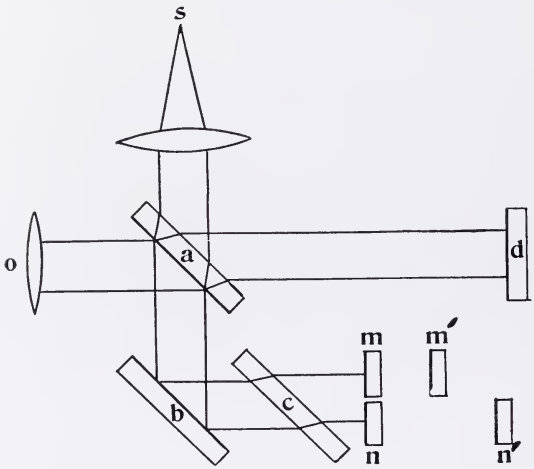


FIG. 72

The two front mirrors of the two standards are adjusted to give fringes in white light with the reference plane. The central fringe in the white-light system is black; the others are colored. Hence we can always distinguish the central fringe. When the central fringes occur in the same relative position upon the two front mirrors m and n, then these two surfaces are exactly in the same plane. Now, if we move the reference plane backward through the length of the shorter standard, its surface will coincide with the mirror m', and at this instant fringes in white light will appear. Thus we have the means of knowing when the reference plane has been moved the length of the first standard to an order of accuracy of one-tenth or one-twentieth of a fringe.

[1] Better, the *image* of d in a and b, which in the figure would coincide with the front surfaces of m and n.

The next process is to move the first standard backward through the same distance. Then the white-light fringes will again appear on the front mirror *m*. Finally we move the reference plane again through the same distance and, if the second standard is twice as long as the first, we get interference fringes on the two rear mirrors of the two intermediate standards. If there is any difference, then the central fringe of the white-light system will not be in the same position on both mirrors, and we shall know that one is twice as long as the other less, say, two fringes, which would mean less one-half micron. In this way we can tell whether one is exactly twice as long as the other or not; and if not, we can determine the difference to within a very small fraction of a wave.

When we multiply the number of waves in the first standard by two, any error in the fractional excess is, of course, also multiplied by two. So the fraction of a wave which must be added to the second number is uncertain. If we observe the fringes produced by one radiation, for example the red, we get a system of circular fringes upon both mirrors of the standard; and if these two systems have the same appearance on the upper mirror as on the lower, then we know this fraction is zero; and the number of waves in the second standard is then the nearest whole number to the number determined. If this is not the case, we can by a simple process tell what the fraction is, and can obtain this fractional excess to any required degree of accuracy. As an example, we may multiply the numbers obtained for the first standard by two, and we find 2,424.74 for the number of red waves in standard No. 2. The correct value of this fraction for red light was found to be .93 instead of .74. Thus the same degree of accuracy which was obtained in measuring the first standard can be obtained in all the standards up to the last. We have thus the means of find-

ing accurately the whole number of waves in the last standard. The whole number obtained by this process of "stepping off" for the red radiation of cadmium was found to be 310,678. The fraction was then determined by the circular fringes, as described above, and found to be .48. In the same way the number for the green radiation was determined as 393,307.93; and for the blue radiation as 416,735.86. To give an idea of the order of accuracy, I would state that there were three separate determinations made at different times and by different individuals, as follows:

Determination	Red	Green	Blue
I	310,678.48	393,307.92	416,735.86
II	310,678.65	393,308.10	416,736.07
III	310,678.68	393,308.09	416,736.02

The fact that these determinations were made at entirely different times, separated by an interval of whole months, and by different individuals, and that we still were able to get, not only the same whole number of waves, but also so nearly the same fractions, gives us a confidence, which we could not otherwise feel, in the possibilities of the process.

In comparing the standards with one another the temperature made no difference, if only it were uniform throughout the instrument, because two intermediate standards side by side, made of the same substance, would expand in exactly the same way, provided we could be sure that both had the same temperature. But in the determination of the number of waves in standard No. 9 it is extremely important to know the temperature with the very highest degree of accuracy. For this purpose some of the best thermome-

ters obtainable were placed in the instrument, and the thermometers themselves were carefully tested, their errors determined, and other well-known precautions taken. In this way the temperature at which the intermediate standard No. 9 contains the number of waves given above was determined to within one-hundredth of a degree.

The final step in the process is the comparison of the decimeter standard with the standard meter. This is a comparatively simple affair. In fact, it is exactly the same as the comparison of the first intermediate standard with the second, except that the second standard is now ten times as long — which necessitates going through the process ten times instead of twice.

Since in this case also we use the fringes for determining when one end of the standard and the reference plane are in the same plane, the error, as before stated, may be as small as one-twentieth of a wave; so that all the errors added together would be of the order of one-half of a wave, or one quarter of a micron.

The conditions which had to be fulfilled by the instrument which was used for this purpose are, then, these: We have, in the first place, to provide for the displacement of the intermediate standard and of the reference plane in such a way that the parallelism of the mirrors is not disturbed. This necessitates that the ways along which the carriage supporting the mirrors moves be exceedingly true. It took a whole month to perform this part of the work — to get the ways so nearly true that there should be no change in the position of the fringes as the mirrors were moved back and forth. In the second place, we must be able to know the position of the mirrors inside of the box which is placed over the instrument to protect it from temperature changes. To secure this, the carriage which holds the mirrors must be moved by means of a long screw carefully calibrated to

within two microns or so. In the third place, since there will be slight displacements, owing to the impossibility of getting the ways absolutely true, it must be possible to correct these displacements. The adjustments for effecting this are shown in Fig. 71. Fourth, we must have a firm support for the longer of the two standards to be compared,

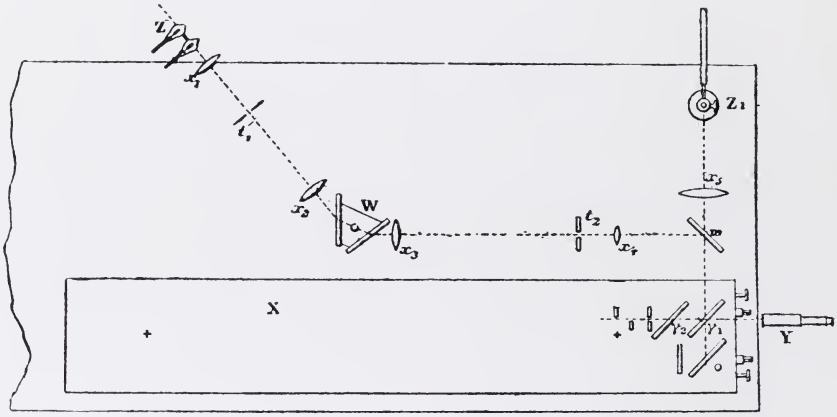


FIG. 73

and a movable support, which moves parallel with itself, for the shorter standard.

The last standard, the auxiliary meter, has to be compared with the standard meter itself, and, therefore, the two must be of similar construction. In other words, in this last comparison we have to resort to the microscope again. For the meter bar which we had in the interferometer itself had two lines upon it as nearly as possible one meter apart, as determined by a rough comparison with the prototype meter. The standard No. 9 had to be compared with this. For this purpose an arm which had a fine mark on it was rigidly fastened to the standard No. 9, and arranged to come in the focus of the microscope. In making this comparison, we must admit, the order of accuracy is not so great.

But there are only two of these to make, so that the possible error is the same as that to which we are liable in comparing two meter bars. This error is unavoidable.

The whole instrument had to be placed in a box, which protected it from temperature changes and drafts of air, and had to be placed on a firm pier so as to keep it as free from

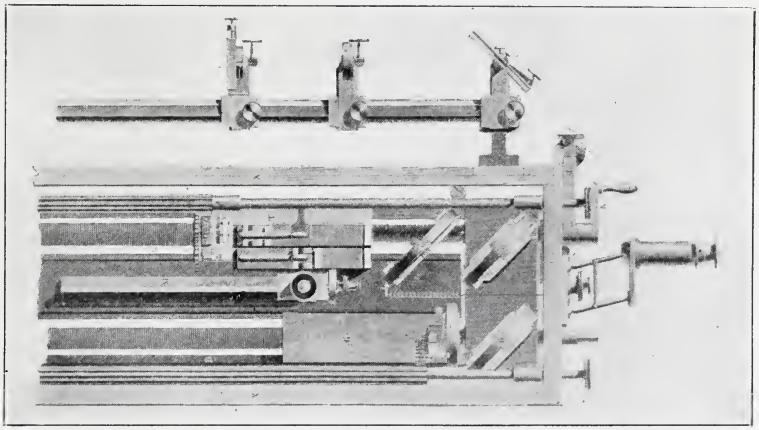


FIG. 74

vibration as possible. Finally, the conditions which have been mentioned above for producing a suitable source of light had to be fulfilled. We have thus a fair idea of what conditions had to be met in constructing the complete apparatus for making this comparison.

We shall now show how these conditions were actually fulfilled in the apparatus that was used for the experiment.

Fig. 73 gives a plan of the entire arrangement. It is easy to recognize the vacuum tube which serves as a source of light and the arrangement of the plates in the interferometer. This arrangement is the same as that shown in Fig. 72. In order to have but one radiation at a time in the instrument, the light from the tube is passed

through an ordinary spectroscope. Thus the light from the tube Z is brought to a focus on the slit t_1. It is then made parallel by means of the lens x_2 and passes through the prism W, which is filled with bisulphide of carbon. The lens x_3 forms the spectral images of the slit t_1 in the plane of the slit t_2. The arm ZW of the spectroscope can be moved so as to bring either the red, the green, or the blue spectral image upon this slit, from which it passes into the instrument.

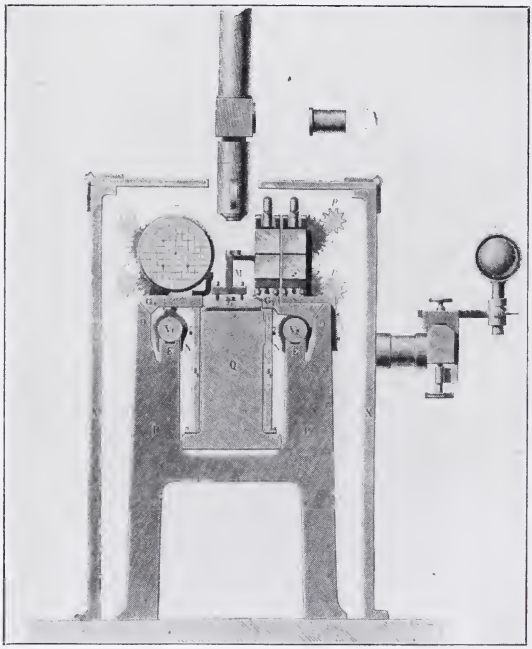


FIG. 75

Fig. 74 is a view of the plan of part of the instrument. The arrangement of surfaces shown diagramatically in Fig. 72 is readily recognized. All of the plates, I may state, instead of being rectangular, have a circular border, because in this form they can be worked true more readily.

Fig. 75 represents a vertical cross-section of the same instrument. It will be noted that the reference plane is divided into sections. This is done in order to enable us to determine very accurately the position of the interference fringes. The two intermediate standards will be recognized at the right.

Fig. 76 represents the actual instrument in perspective. In this the two microscopes, with their arrangement for producing an illumination on the meter bar by means of reflected light, are shown. On the left are the handles which turn the two screws. One of these moves the intermediate standard

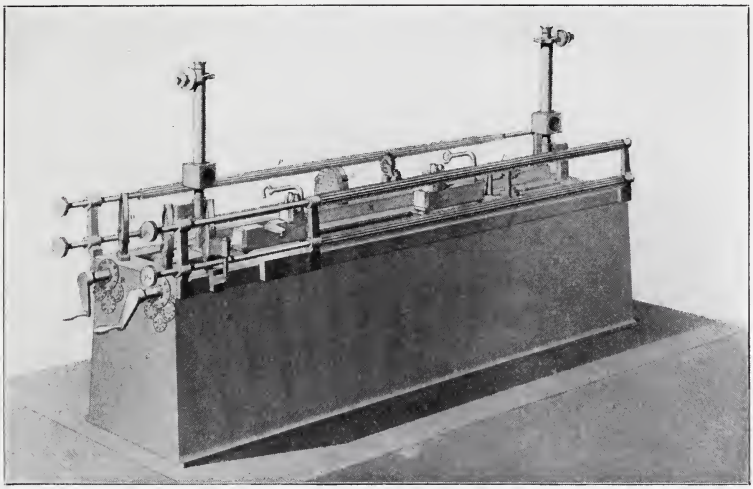


FIG. 76

and the other moves the reference plane. The complete instrument in the case which protects it against temperature changes is shown in Fig. 77.

This investigation was reported in the spring of 1892 to Dr. Gould, who at that time represented the United States in the International Committee of Weights and Measures. It was principally through his goodness that I was asked to carry out the actual experiments at the International Bureau of Weights and Measures at Sèvres. Many of the accessories that were required for the instrument which has just been described had to be made in this country, and were

taken over and installed in one of the laboratories of the Bureau.

The standard meter itself is kept in a vault underground and under double lock and key, and is inspected only once in ten years, and even then it is not handled any more than

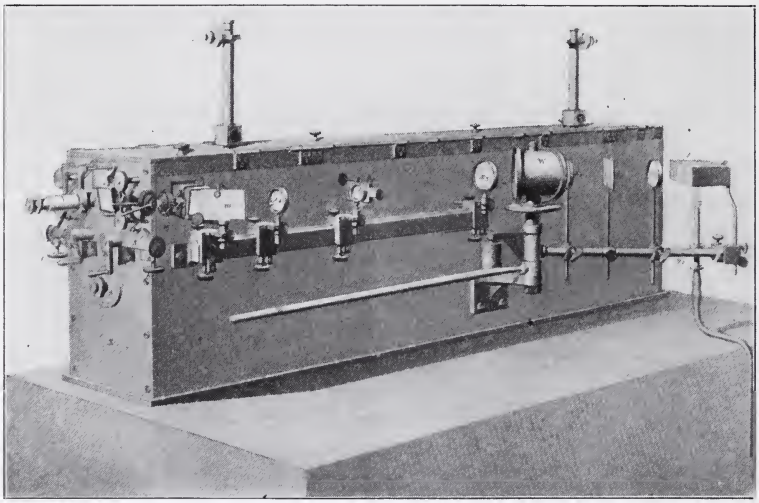


FIG. 77

is absolutely necessary. It took the better part of an entire year to accomplish the work as it has been described. The final result of the investigation was that the number of light waves in a standard meter was found to be, for the red radiation of cadmium 1,553,163.5, for the green 1,966,249.7, for the blue 2,083,372.1—all in air at 15° C. and at normal atmospheric pressure.

It is also worth noting that the fractions of a wave are important, because, while the absolute accuracy of this measurement may be roughly stated as about one part in two million, the relative accuracy is much greater, and is probably about one part in twenty million.

The question may be asked: What is the object of making such determinations, when we know that the standard itself would not change by any amount which would vitiate any ordinary measurements? The reply would be that, while the care taken of the standards is pretty sure to secure them from any serious accident, yet we have no means of knowing that any of these standards are not going through some slow process of change, on account of a gradual rearrangement of the molecules. Now that we have compared the meter with an invariable standard, we have the means of detecting any slow change and of correcting the standard which has been vitiated by such process. Thus it is now possible to control, by reference to the standard light waves, the standard of length. The standard light waves are not alterable; they depend on the properties of the atoms and upon the universal ether; and these are unalterable. It may be suggested that the whole solar system is moving through space, and that the properties of ether may differ in different portions of space. I would say that such a change, if it occurs, would not produce any material effect in a period of less than twenty millions of years, and by that time we shall probably have less interest in the problem.

SUMMARY

1. We find that three propositions for expressing our standard of length in terms of some invariable length in nature have been made, namely:

a) Measurement of the seconds pendulum.

b) Measurement of the earth's circumference.

c) Measurement of light waves.

The first two, as well as the first plan proposed for carrying out the third, *i. e.*, the method of the diffraction grating, have been found deficient in accuracy.

2. The second or interference method of utilizing light

waves, while ideally simple in theory, necessitates in practice an elaborate and complicated piece of apparatus for its realization. But, notwithstanding the delicacy of the operation, it is capable of giving results of such extraordinary accuracy that, were the fundamental standard lost or destroyed, it could be replaced by this method with duplicates which could not be distinguished from the originals.

LECTURE VI

ANALYSIS OF THE ACTION OF MAGNETISM ON LIGHT WAVES BY THE INTERFEROMETER AND THE ECHELON

A LITTLE over a year ago the scientific world was startled by the announcement that Professor Zeeman had discovered a new effect of magnetism on light. The experiment that he tried may be briefly described in the following way: If we place a sodium flame in front of the slit of a spectroscope, we get in the field of view a bright double line. If the flame is placed between the poles of a powerful electro-magnet, it is found that the lines are very much broadened; at least this was the way in which the announcement of the discovery was first made. It may be mentioned that a somewhat similar observation was made by M. Fievez a long time before. He found that the sodium lines in the spectrum were modified by the magnetic field, but not quite in the way that Zeeman announced; instead of the lines being broadened, he thought that each separate sodium line was doubled or quadrupled. It seems that, long before this, the experiment had actually been tried by Faraday, who, guided by theoretical reasons, conjectured that there should be some effect produced by a powerful magnetic field upon radiations.

The only reason why Faraday did not succeed in observing what Fievez and Zeeman observed afterward was that the spectroscopic means at his disposal at the time were far from being sufficiently powerful. The effect is very small at best. The distance between the sodium lines being taken as a kind of unit for reference, the separate sodium lines, as was shown in a preceding lecture, have a width of about one-hundredth of the distance between the two. The broadening, or

doubling, or other modification which is produced in the spectrum by the magnetic field, is of the order of one-fortieth, or perhaps one-thirtieth, of the distance between the sodium lines. Hence, in order to see this effect at all, the highest spectroscopic power at our disposal must be employed. Subsequent investigation has shown, indeed, that still other modifications ensue, which are very much smaller even than this, and which cover a space of perhaps only one-hundredth to one hundred-and-fiftieth of the distance between the sodium lines. They are, therefore, beyond reach of the most powerful spectroscope.

It occurred to me at once to try this experiment by the interference method, which is particularly adapted to the examination of just such cases as this, in which the effect to be observed is beyond the range of the spectroscopic method. The investigation was repeated in very much the same way as described by Zeeman, namely: A little blow-pipe flame was placed between the poles of a powerful electro-magnet; a piece of glass was placed in the flame to color it with sodium light. The light, instead of passing into the spectroscope, was sent into an interferometer and analyzed by the method described in Lecture IV. The visibility curves which were thus obtained showed that, instead of a broadening, as was first announced by Zeeman, each of the sodium lines appeared to be double. The visibility curves which were observed are shown in Fig. 78, and in Fig. 79, the curves which give the corresponding distribution of the light in the source. In the former figure the vertical distances of the different curves represent the clearness of the fringes, and the horizontal distances the differences in path. In curve *A*, as the difference in the paths increases, the fringes become less and less distinct, until at forty millimeters the fringes have almost entirely disappeared. This curve represents the visibility of the sodium

flame without any magnetic field. The corresponding intensity curve *A* (Fig. 79) shows that the center of the line has the greatest intensity and that the intensity falls off rapidly on either side, the width of the line corresponding to something like one-hundredth of the distance between the two sodium lines. When the field was created by simply closing the current through the magnet, the visibility curve assumed the form indicated in curve *B*. The corresponding distribution of light is shown in the second of the intensity curves, *B* (Fig. 79) and we see that the line shows simply a broadening, with a possible indication of doubling. The field was then increased considerably; curve *C* (Fig. 78) represents the visibility. The corresponding intensity curve shows clearly that the line is double. The other curves were obtained by increasing the field gradually, and it will be noted that the result is an increasing separation of the line and, at the same

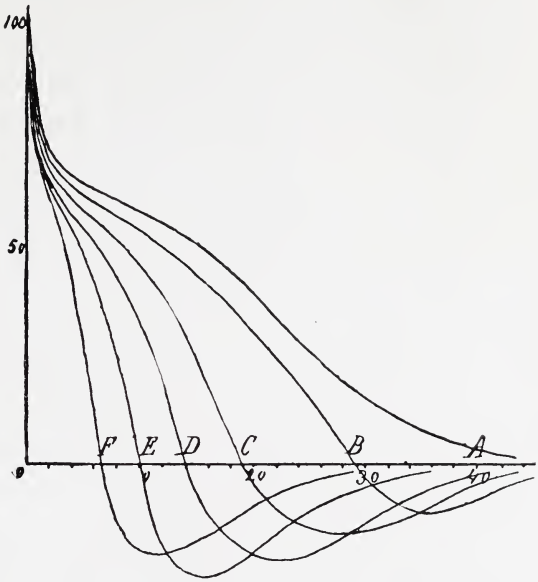


FIG. 78

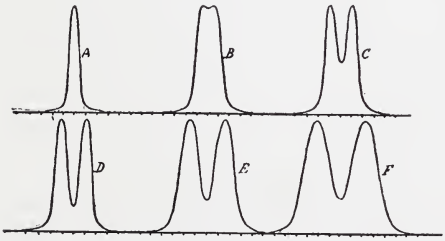


FIG. 79

time, a considerable broadening out of the two separate elements.

This same experiment was tried with other substances, especially with cadmium, and it was found that almost identical results were obtained with cadmium light as with sodium. It was therefore inferred that the observations announced by Zeeman were, at any rate, incomplete, and it was thought that possibly the instruments at his command were not sufficiently powerful to show the phenomena of the doubling. Shortly after this experiment was published another announcement was made by Zeeman. In this he states that there is not simply a broadening of the lines, but a separation of them into three components, and, what was very much more interesting, that these three components are polarized in directions at right angles with each other: the middle line polarized in one plane and the two outer lines in another.

To make the meaning of this clear, we shall have to make a brief digression into the subject of the polarization of light. It will be remembered that in one of the first illustrations of wave motion light waves were compared with the waves along a cord, and it was stated that the vibrations which caused the phenomena of light are known to be vibrations of this character rather than of the character of sound waves. The sound waves consist of vibrations in the direction of the propagation of the sound itself. The motion of the particles in the light waves are at right angles to their direction of propagation. These transverse vibrations, as they are called, may be vertical or horizontal, or they may be diagonal, or they may move in a curved path, for instance in circles or ellipses.

In the case of ordinary light the vibrations are so mixed up together in all possible planes that it is impossible to separate any one particular vibration from the rest without

special devices, and such devices are termed "polarizers." They may be likened very roughly to a grating the apertures of which determine the plane of vibration. Through such a grating we can transmit vibrations along a cord only in the plane of the apertures. A vibration at right angles to this plane will not travel along the cord beyond the grating. The corresponding light phenomena may be illustrated by attempting to pass a beam of light which has been polarized through a medium which acts toward the light waves as does the grating toward the waves on the cord. It is found that crystals act as such media. Thus a plate of tourmaline possesses this property. For, as is well known, if two plates of tourmaline be placed so that their optical axes are parallel with each other, almost as much light will pass through the two as through either one alone. But if the axes are set at right angles to each other by turning one of them through 90°, the light is entirely cut off. Turning again through 90°, the light again appears, etc. In the case of the tourmaline the vibrations which have passed through one plate are all in one plane.

There is another important case in which the light is said to be polarized, namely, when the motion of the particles is circular. We may have two such circular vibrations — one in which the motion is in the direction of the hands of a watch, called right-handed, and the other in which the motion is in the direction opposite to that of the hands of a watch, and which is therefore called left-handed. We may consider that each one of these vibrations is compounded of two plane vibrations of equal intensity, in one of which the motion is horizontal and in the other vertical, and which differ from one another in phase, this difference being one-fourth of a period for the left-handed and three-fourths of a period for the right-handed. If we add together two such circular vibrations of equal intensity, their horizontal components would exactly

neutralize each other, so that there would be no horizontal motion at all. The vertical components, however, being in the same direction, will add to each other, so that the resultant of two beams of light polarized circularly in opposite directions and of equal intensity is a plane polarized ray.

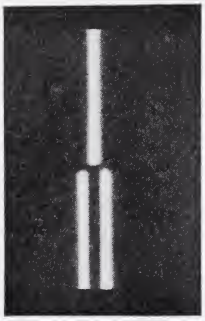


FIG. 80

To return, now, to Zeeman's phenomenon. Fig. 80 represents one of the sodium lines when examined in a direction at right angles to the magnetic field. The upper line represents the appearance when the light is polarized so that only horizontal vibrations reach the spectroscope. If, however, the polarizer is rotated through 90°, so that only vertical vibrations pass, the appearance is that of the lower half of the diagram, the two side lines appearing and the central line disappearing. Finally, if the light is examined in the direction of the magnetic field, which can be accomplished by boring a hole through the pole of the magnet, it is found that only two are visible — the two outside ones; and one of these is composed of light which vibrates circularly in the direction of the hands of a watch, and the other is circularly polarized in the opposite direction.

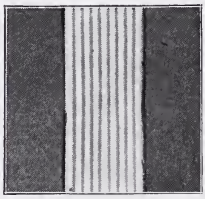


FIG. 81

An extremely beautiful and simple explanation of this phenomenon has been given by Lorentz, Larmor, Fitzgerald, and a number of others. At the risk of introducing a few technicalities, I will venture to repeat this explanation in a simple form. For this purpose it is necessary to know that the particles or atoms of matter are each supposed to be associated with an electric charge, and that such a charged par-

ticle is termed an "electron." This hypothesis, made long before Zeeman made his discovery, was found necessary to account for the facts of electrolysis. For the decomposition of an electrolyte by an electric current is most simply explained upon the hypothesis that it contains positively and negatively charged particles, and that the positively charged atoms go toward the negative pole, and the negatively charged toward the positive pole. They then give up their electricity, and this giving up of electricity constitutes an electric current. Hence this assumption, which is useful in explaining the Zeeman effect, is nothing new. It is known, also, that the vibrations of these particles, or of their electric charges, produce the disturbance in the ether which is propagated in the form of light waves; and that the period of any light wave corresponds to the period of vibration of the electric charge which produces it.

The most general form of path of such a vibrating electric charge would be an ellipse. Now, an elliptical vibration can always be resolved into a circular vibration and a plane one, so that any polarized ray may be resolved into a plane polarized ray and a circularly polarized ray. So all we need to consider are plane and circularly polarized rays. But we may suppose that a plane vibration is due to two oscillations in a circle, one going in a direction opposite to that of the hands of a watch, and the other in their direction. Hence, we need consider only circular vibrations. Now, if the electric charge is moving in a circle, it can be shown that when the plane of the circle is at right angles to the direction between the two poles of the magnet, the effect of the field would be to accelerate the motion when the rotation is, say, counter-clockwise, but to retard it when it is clockwise.

It was shown above that the position of a spectral line in the spectrum depends on the period of the light which produces it. Hence the position of the line will be altered

when any current is passing about the electro-magnet. When the current is passing in a certain direction, the velocity of rotation of the particles moving, say counter-clockwise, is increased. Hence the period of vibration is smaller; *i. e.*, the number of vibrations, or the frequency, is greater. In this case there will be a shifting toward the blue end of the spectrum by an amount corresponding to the amount of the acceleration. Those particles which are rotating in an opposite direction, *i. e.*, clockwise, will be retarded, the frequency will be less, and the spectral lines will be shifted toward the red. These two shiftings would account, then, for the double line. It is further clear that those vibrations which occurred in planes parallel to the lines of force of the magnetic field would be unaltered. These vibrations would then produce the middle line, which is not shifted from its position by the magnetic field.

Again, if we are viewing the light in a direction at right angles to the lines of force of the field, the vibrations of those particles which are affected by the field would have no components parallel to the field. If the particles are revolving in a plane perpendicular to the field, then, when viewed in this direction, they would appear to be moving only up and down; *i. e.*, they would send out plane polarized light whose vibrations are vertical. These two vertical vibrations form the two outer lines of the triplet, and it can be shown that the light is plane polarized by passing it through a polarizer. Those particles which are vibrating horizontally do not have their period of vibration altered by the field. Consequently we get a single line whose position in the spectrum is not changed, and which is plane polarized in a plane at right angles to that of the other two.

When this second announcement of Zeeman appeared, it seemed worth while to repeat the experiments with the

interferometer, especially as it was pointed out that probably the reason why a single or a double line appeared, instead of a triple line, was because part of the light corresponding to the middle line was cut off by the reflection from the separating plate of the interferometer. The light thus reflected is polarized, and most of the light which should have formed the central image is thus cut off. It was therefore determined to repeat these experiments under

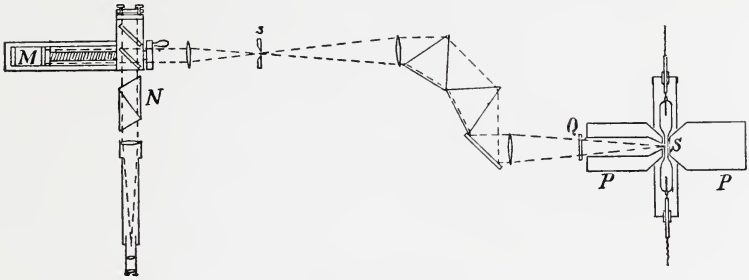


FIG. 82

such conditions that we could be perfectly sure that light which reached the interferometer vibrated in only one plane. To accomplish this it is necessary merely to introduce a polarizer into the path of the light.

Fig. 82 represents the arrangement of the experiment with the interferometer. The source of light, instead of being sodium in a Bunsen flame, is vapor in a vacuum tube, illuminated by an electric discharge. The capillary part of the tube is placed between the poles of the magnet.

The light is first passed through an ordinary spectroscope, so that there is formed at *s* a spectrum, any part of which we may examine. The slit at *s* allows only one radiation to pass into the interferometer. Thus, if we examine cadmium light, we may allow the red to pass through, or the green, or the blue. The light is made parallel by a lens and then passes into the interferometer. The arrangement for

examining separately the vertical vibrations alone and the horizontal vibrations alone is represented at N, and consists merely of a Nicol prism which can be rotated about a horizontal axis.

With this arrangement a different set of visibility curves was obtained. These are shown in Figs. 83, 84, 85.

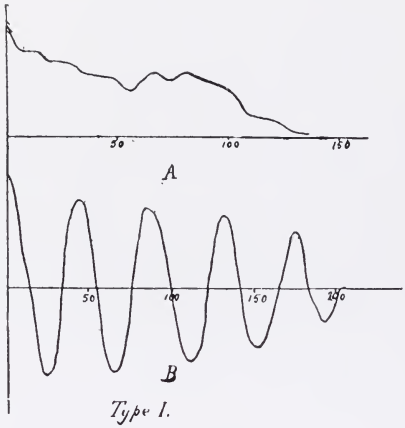


FIG. 83

The upper curve of Fig. 83 represents the visibility curve produced by the horizontal vibrations of the red cadmium light in a strong magnetic field. For the vertical vibrations the visibility curve is something totally different, and is shown in the lower half of the figure. The effect of the field is readily appreciated by comparing this figure with Fig. 66, which corresponds to the red cadmium line without any magnetic field.

The upper curve of Fig. 84 represents the visibility curve of the blue cadmium vapor when the horizontal vibrations only are allowed to pass through. When vertical vibrations only are allowed to pass through, the curve has the form shown in the lower half of the figure.

The case of the green radiation, when there is no field, is shown in Fig. 67 above. When the magnetic field is on, and when the horizontal vibrations only are allowed to pass through, the visibility curve has the form of the upper curve in Fig. 85. When vertical vibrations are allowed to pass through, it has the form of the lower curve.

The intensity curves corresponding to Figs. 83, 84, and 85 are shown in Fig. 86. The upper three correspond to

the horizontal vibrations, while the lower three correspond to the vertical vibrations. In the case of the red radiations it will be noted that, whether there is a magnetic field or not, there is no particular change for red cadmium light when the horizontal vibrations alone are considered. When the field is on, the vertical vibrations give a double line, or possibly one of more complex form.

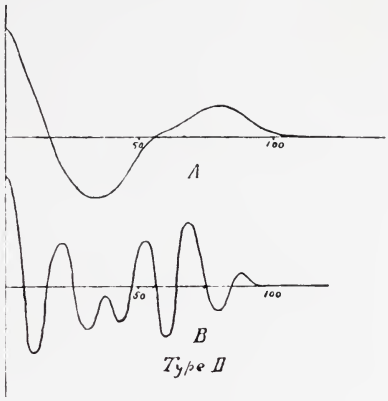


FIG. 84

In the case of the blue radiations, however, when there is a magnetic field and only horizontal vibrations are allowed to pass through, the line is double. The doubling is very distinct, and the separation is so wide that it should be easily seen by means of the spectroscope. When the vertical vibrations alone are allowed to pass through, there is a very much more complicated effect. In all cases we can see that the line is double, as in the case of red cadmium light, but in this case each component of the double lines is at least quadruple, or even more complex.

In the case of the green radiation, when horizontal vibrations only are considered, we have a triple line for the central line of the Zeeman triplet. When horizontal vibrations alone are allowed to pass through without a magnetic field, it resembles in general character the red line (*cf.* Fig. 67). When vertical vibrations are examined in the magnetic field, the line is highly complex; and in this case it is absolutely certain that each of the components of the double consists of at least three separate lines. The phenomenon is perfectly symmetrical about the central line.

It appears from these results that the Zeeman effect is a much more complex phenomenon than was at first supposed, and therefore the simple explanation that was given above no longer applies. At any rate, it must be very seriously modified in order to account for the much more highly complex character of the phenomena, as here described. The complete theory has not yet been worked out, and meanwhile we must gather whatever information we can concerning the behavior of as many different radiations as possible. Every attempt to deduce some general law which will cover all cases at present known has thus far proved unsuccessful. There are a number of anomalies which seem even more difficult to account for than the doubling of this middle line and the multiplication of the side lines. For example, in one of the radiations examined, the line without any magnetic field appeared as quadruple, but when the magnetic field was on, it appeared as a single line.

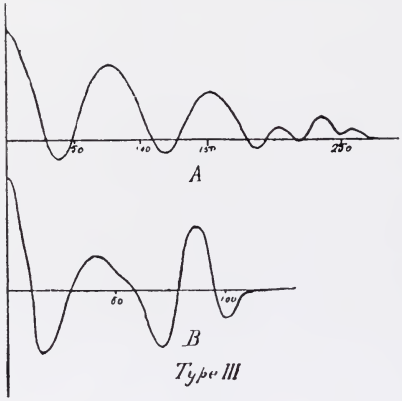


FIG. 85

There are quite a number of other interesting cases, which we have not time to consider now. The explanation of these anomalies will probably not be given until long after the explanation of the doubling and tripling and multiplication of separate lines.

The examination of spectral lines by means of the interferometer, while in some respects ideally perfect, is still objectionable for several reasons. In particular, it requires a very long time to make a set of observations, and we can

examine only one line at a time. The method of observation requires us to stop at each turn of the screw, and note the visibility of the fringes at each stopping-place. During the comparatively long time which it takes to do this the character of the radiations themselves may change. Besides, we have the trouble of translating our visibility curves into distribution curves. Hence it is rather easy for errors to creep in.

On account of these limitations of the interferometer method, attention was directed to something which should

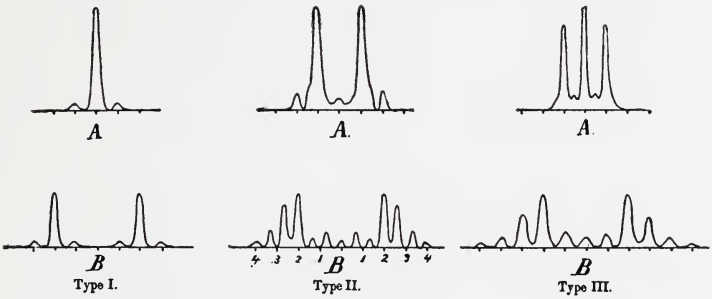


FIG. 86

be more expeditious, and the most promising method of attack seemed to be to try to improve the ordinary diffraction grating. The grating, as briefly explained in one of the preceding lectures, consists of a series of bars very close together, which permit light to pass through the intervals between them. The first gratings ever made were of this nature, for they consisted of a series of wires wound around two screws, one above and one below. This first form of grating answered very well for the preliminary work, but is objectionable because the interval between the wires is necessarily rather large, *i. e.*, the grating is rather coarse. If we allow light to pass through these intervals, each interval may be considered to act as a source of light.

From each of these sources it is spread out in circular waves. If the incident wave is plane and falls normally upon the grating, all these waves start from the separate openings in the same phase of vibration. Hence, in a plane parallel to the grating we should have, as the resultant of all these waves, a plane wave traveling in the direction of the normal to the grating. When this wave is concentrated in the focus of a lens, it produces a single bright line, which is the image of the slit and is just as though the grating were not present.

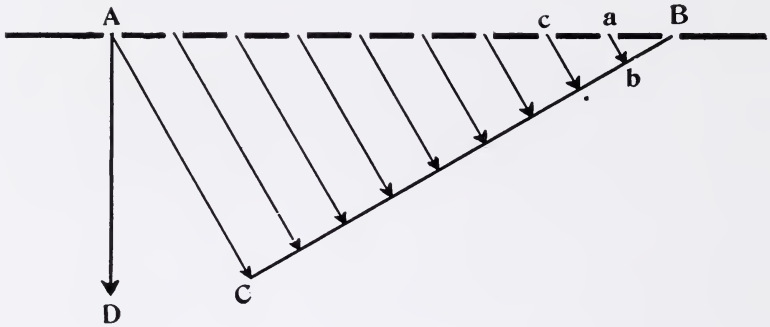


FIG. 87

Suppose we consider another direction, say AC (Fig. 87). We have a spherical wave, starting from the point B, another in the same phase from the point a, etc. Now, if the direction AC is such that the distance ab from the opening a to the line through B perpendicular to AC is just one wave, then along the line BC the light from the openings B and a differ in phase by one whole wave. When ab is equal to one wave, cd will be equal to two waves; hence, along BC the light from the opening c will be one wave behind the light from a, etc.; and if these waves are brought to a focus, they will produce a bright image of the source. Since the wave lengths are different for different colors, the direction AC in which this condition is fulfilled will be different for different colors. A grating will there-

fore sort out the colors from a source of light and bend them at different angles, forming a spectrum. Since the blue waves are shorter than the red, the blue will be bent least and the red most, the intervening colors coming in their proper order between. Again, we may also have an image formed when the direction AC is such that this difference in phase of the light from successive openings, instead of one wave, is two. The spectrum thus formed is said to be of the second order. When this difference in phase is three waves, the spectrum is said to be of the third order, etc.

Plate I, Fig. 2, represents the spectrum produced by a coarse grating. The source of light was a narrow slit illuminated by sunlight. The central image appears just as though no grating were present, and on either side are diffuse spectral images colored as on Plate I. Three such images, which are the spectra of the first, second, and third orders, may be counted on the right, and the same on the left. The grating used in producing this picture had about six hundred openings to the inch. Now, a finer grating produces a much greater separation of the colors. The large concave gratings used for the best grade of spectroscopic work produce spectra of the first order which are four feet long. Those of higher order are correspondingly longer.

The efficiency of such gratings depends on the total difference of path in wave lengths between the first wave and the last. Thus in the grating shown in Fig. 87 there will be, in the case of the first spectrum, as many waves along AC as there are openings between A and B. If we call the total number of openings in the grating n, then there will be n waves along AC. In the second spectrum, then, since each one of the intervals corresponds to two waves, the total difference in the path is twice as great, so that the number of waves in AC will be $2n$. For the third spectrum the number would be $3n$, and for the mth spectrum mn.

The efficiency of the grating depends on the order m of the spectrum and the number n of lines in the grating, *i. e.*, on the product of the two. Hitherto the efforts of makers of gratings have been directed toward increasing n as much as possible by making the total number of lines in the grating as great as possible. It has been found that as many as 100,000 lines can be ruled side by side on a metallic surface; but in ruling 100,000 lines it is extremely difficult

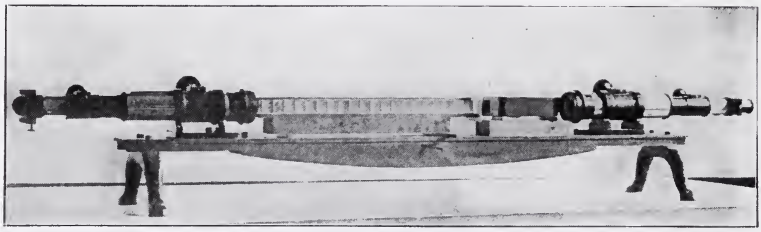


FIG. 88

to get them in their proper position. Very little attention has as yet been directed toward producing a spectrum of a very high order. The chief reason for this is that the intensity of the light in the spectra of higher orders diminishes very rapidly as the order increases. The first spectrum is by far the brightest; the second has an intensity of something like one-third of the first, and the succeeding spectra are still fainter. There have been, occasionally, gratings in which the diamond point happened to rule in such a way as to throw an abnormal proportion of light in one spectrum. Such are exceedingly rare and exceedingly valuable. It seems to be a matter of chance whether the diamond rules such gratings or not. It was with the double purpose of multiplying the order of the spectrum, and at the same time of throwing all the light in one spectrum, that the instrument shown in Fig. 88 was devised.

The method of reasoning which led to the invention of this instrument may be of interest. We will suppose that, in order to throw the light in one spectrum, the diamond point could be made to rule a grating with a section like that shown in Fig. 89, the distance between the steps being exactly equal and the surfaces of the grooves perfectly polished. Suppose that the light came in the direction indicated nearly normal to the surface of the groove. The light would be reflected back in the opposite direction, and that which came from each successive groove would differ in phase from that from the adjacent grooves by a number of waves corresponding to double the difference in path. The retardation, instead of being one wave, would be twice the number of waves in this distance. If the distance between the grooves were very large, the number of waves in this distance would also be very large, so that the order of the resulting spectrum would be correspondingly high. Further, almost all the light returns in one direction, so that the spectrum we are using will be as bright as possible.

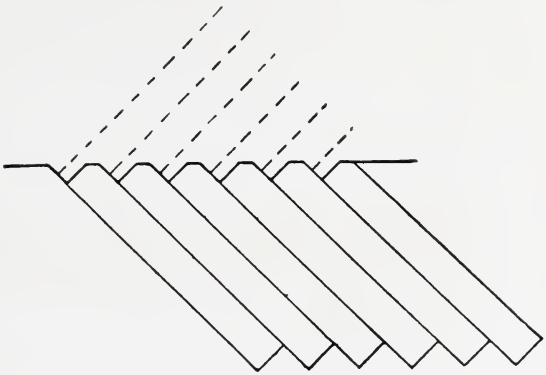


FIG. 89

We have thus shown, at least theoretically, the possibility of producing a very high order of spectrum, and at the same time of getting almost all the light in one spectrum. However, the necessary condition is that the distances between the grooves be equal within a very small fraction of a light wave. This is a difficult, but not a hopeless, problem. In fact, we may obtain the desired retardation

by piling up plates of glass of the same thickness. These plates of glass can be made originally of a single piece, as nearly uniform in thickness as possible. It has been possible to obtain plates, plane parallel, so accurate that the thickness was the same all over to within one-hundredth of a light wave; that is, less than one five-millionth of an inch. If we could place a number of such plates in contact with each other, we should have the means of producing any desired retardation of light reflected from one surface over the light reflected from the next nearest surface, and should be able to make this retardation exactly the same number of waves for all the intervals. The difficulty lies in the fact that we cannot place the plates in contact even by applying a pressure large enough to distort the glasses, because of dust particles. The thickness of such particles is of the order of a light wave. It is therefore difficult to get the plates much closer together than about three waves. If this distance were constant, no harm would be done, but it varies in different cases; so the extreme accuracy of the thickness of the glass is practically valueless.

Fortunately there is a way of getting around the difficulty, and this way has, at the same time, other advantages. Suppose that, instead of reflecting the light from such a pile of glass plates, we allow it to go through. The light travels more slowly in glass than in air—in the ratio of one and one-half to one—and the retardations produced by the successive plates in the light which has passed through are now exactly the same. In this way it has been found possible to utilize as many as twenty or thirty of such plates, and the retardation produced by each plate would correspond to the difference in the optical path between a layer of air and an equally thick layer of glass. Some of these plates have been made as thick as one inch. Roughly speaking, there are 50,000 waves in an inch of air; the number in an equal thickness of

glass would be one and one-half times as great, so that the difference in path would be 25,000 waves. But the resolving power is the order of spectrum multiplied by the number of plates. If we are observing, therefore, in the 25,000th spectrum, and there are thirty such plates, the resolving power would be 750,000; whereas the resolving power of the best gratings is about 100,000.

There are, however, disadvantages in the use of this instrument. One of these may be illustrated as follows: Suppose we take the case of the ordinary grating; the first spectral image is rather widely separated from the central image of the slit, the second spectral image is twice as far away as the first, and the third spectral image will start three times as far away as the first, and will also be three times as long. The result is that parts of the second and third overlap. The overlapping becomes greater and greater as the order of the spectrum increases, so that when the 25,000th spectrum is reached the spectra are inextricably confused. Where we have to deal with a few simple radiations, however, as in cadmium or sodium, this overlapping is not so serious as might be supposed. We have a very simple means of getting rid of the worst of it by analyzing the light by means of a prism before it enters the pile of plates.

The construction of the instrument is not very different from that of the ordinary spectroscope. The light passes through a slit and then through a lens, by which it is made parallel. It then passes through the pile of plates—the echelon, as it has been named—and into the observing telescope. With this instrument the results obtained by the method of visibility curves have been confirmed. Thus Fig. 81 shows the appearance of the green mercury line in the field of view of the echelon when the source is in a strong magnetic field. In the three central components the vibra-

tions are horizontal, while in the outer three on both sides the vibrations are vertical. An idea of the power of this instrument can be obtained by comparing Fig. 81 with Fig. 80, which gives the appearance of the line as seen in the best grating spectroscope.

SUMMARY

1. The investigation of the changes produced in the radiations of substances by placing them in the magnetic field is in general a phenomenon barely within the range of the best spectroscopes, and there are some features of it which it would be entirely hopeless to attack by this method.

2. Such investigations, however, are precisely the kind for which the interference method is particularly adapted. In fact, the results of the investigation by the method of visibility curves have furnished a number of new and interesting developments which could only with difficulty have been obtained by the ordinary spectrometer methods.

3. Fertile as this method has shown itself to be, there are, nevertheless, a number of serious drawbacks. In order to obviate these a new instrument was devised, the echelon spectroscope, which has all the advantages of the grating spectroscope, together with a resolving power many times as great. With the aid of this instrument all the preceding deductions have been amply verified and a number of new and interesting facts added to the store of our knowledge of the Zeeman effect.

LECTURE VII

APPLICATION OF INTERFERENCE METHODS TO ASTRONOMY

Our knowledge of the heavenly bodies is still very limited. The little that we have learned has been acquired almost entirely with the assistance of the telescope, or the telescope compounded with the spectroscope. Without these, the stars and the planets would always remain, even to the most perfect unaided vision, as simple points of light. With these aids we are every year adding very much to our knowledge of their constitution, their form, their structure, and their motions. For example, the spectroscope gives information concerning the elements contained in the sun and the stars; for by means of the dark or bright lines in the spectrum we are able to identify elements by the position of their spectral lines, and from this identification we are able to infer, with almost absolute certainty, the presence of the corresponding material in the heavenly body which is examined. The same is true of comets and nebulæ. By the general character of the spectrum we may also distinguish whether these bodies are in the form of incandescent gases, or whether they are in solid or liquid form; and we can, to a certain extent, infer their temperature. We can even determine whether the body is approaching or receding. For example, if the body is approaching, the waves are crowded together so that their wave length will be shortened, and hence they have a correspondingly altered position in the spectrum, *i. e.*, the line will be shifted toward the blue end of the spectrum. If the body is receding, the spectral line is shifted in position toward the red end of the spectrum.

By the telescope we have discovered that all the planets, including many of the minor planets, have discs of appreciable size. We have found markings on the planets, have discovered the satellites of Jupiter and the rings of Saturn, and have observed various interesting details concerning the structure of these rings. The strange markings on the planet Mars, which bear such a remarkable resemblance to the works of intelligent beings, are among the most interesting of the recent revelations of the telescope.

It is hard to realize that such observations concern bodies that are distant millions of miles from us; in fact, the distance is so great that it can be more readily expressed by the time light takes to reach us from these bodies. In some cases this may be as much as several years. We can compare this distance with the circumference of the earth, by considering that light or a telegram will go around the earth seven times in a second, while from these bodies it would take several hours for light to reach us. Yet these are our nearest neighbors, or, rather, members of our immediate family. Our farther neighbors are so remote that probably the light from many of them has not yet reached us. To these more distant bodies our own little family of planets is probably invisible; even the sun itself is a second-rate star. If, however, Jupiter were sufficiently bright, then the sun and Jupiter together would form what is called a "double star," and to an inhabitant of a distant planet which might be traveling about this distant star it would appear as a double star with a separation of about one second, which may be expressed as the angle subtended by two luminous points about one-half inch apart when at a distance of three miles. They would therefore be entirely invisible to the naked eye as separate objects.

One of the most serious difficulties in the way of further progress in the investigation of the telescopic characteristics

of the planets and of the constitution of star systems, is what is called bad "seeing." It must be remembered that light, in order to reach a telescope, must pass through from forty to one hundred miles of atmosphere. This atmosphere is not homogeneous. If the atmosphere were homogeneous, there would not be any very serious objection. The intensity of the light from the object would be practically as great as if there were no air present. But the air is unequally heated, and therefore has unequal densities in different portions. Hence the different portions of a beam of light which have passed through different parts of the atmosphere and reached different parts of the objective of the telescope would be differently retarded, and these differences in retardation would not be constant, but would vary, sometimes rapidly and sometimes slowly, producing what is technically called "boiling."

This unsteadiness of the image is the most serious difficulty with which astronomers have to contend; there is no instrumental remedy. The best that can be done is to choose an appropriate site, and it seems to be the general opinion of astronomers that such a site is best chosen on some very high plateau or tableland. By some it is considered that a high mountain top is a desirable location, and there is no question that such a site possesses very marked advantages in consequence of the rarity of the air. If the air were very rare, "boiling" would have less effect than it has in dense air. But to compensate this advantage we have the very bad effect of currents of heated air traveling up the side of the mountain. As a matter of fact, however, even in the worst locations, there are occasional nights when the astronomer has almost perfect seeing — when even the largest instruments attain almost their theoretical limit of accuracy. This theoretical efficiency may be most conveniently tested by observations on double stars.

The resolving power, as shown in one of the preceding lectures, depends on the size of the diffraction rings which are produced about the image of a star. It was also shown that the smallest angle which a telescope could resolve was that subtended at the center of the lens by the radius of the first dark ring, and this angle is equal to the ratio of the length of the light wave to the diameter of the objective. For example, if we consider a 4-inch glass, the length of the light wave being $\frac{1}{50000}$ of an inch, this angle would be $\frac{1}{200000}$. If the lens were a 40-inch glass, the angle would be something like $\frac{1}{2000000}$, which can be represented by the angle subtended by a dime at the distance of fifteen miles. Hence, if we had two such dimes placed side by side, the largest glass would scarcely separate them.

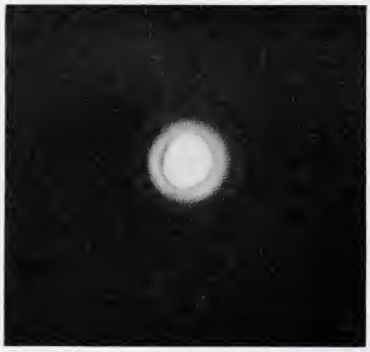


FIG. 90

Fig. 90 is an actual photograph of the image of a point of light taken with an aperture smaller than that of a telescope, but otherwise under the same conditions under which a telescope is used. It is easy to see that, surrounding the point of the image, there is a more or less defined white disc, and beyond this a dark ring. Outside of this dark ring there are a bright ring and another dark ring. Theoretically, there are a great number of those rings; practically, we see only one or two under the most favorable conditions.

This figure represents the appearance of the image of one of Jupiter's satellites as it would be observed in one of the largest telescopes under the most favorable conditions. If it be required to measure the diameter of one of these very

distant objects, a pair of parallel wires is placed as nearly as possible upon what is usually called the edge of the disc, as shown in Fig. 91. The position of this edge varies enormously with the observer. One observer will suppose it well within the white portion; another, on the edge of the black portion. Then, too, the images vary with atmospheric conditions. In the case of an object relatively distinct there may be an error of as much as 5 to 10 per cent. In many cases we are liable to an error which may amount to 15 per cent., while in some measurements there are errors of 20 to 30 per cent.

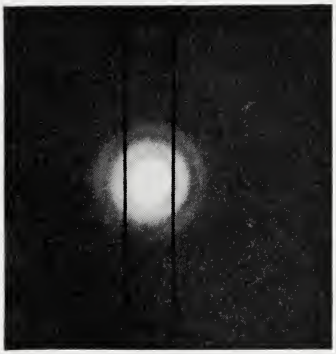


FIG. 91

Suppose the object viewed were a double star. In general, the appearance would be very much like that represented in Fig. 92, except that, as before stated, in the actual case the appearance would be troubled by "boiling." It will be noted that as long as the diffraction rings are well clear of each other we need not have the slightest hesitation in saying that the object viewed is a double star.

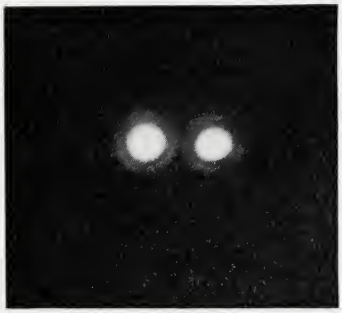


FIG. 92

Fig. 93 represents under exactly the same conditions two points, artificial double stars, but very much closer together. In this case the diffraction rings overlap each other. It will be seen that the central spot is elongated, and the expert

astronomer may decide that the star is double. This elongation can under favorable circumstances be detected even a considerable time after the diffraction rings merge into each other. If the atmospheric conditions were a little worse, such a close double would be indistinguishable from the single star, and if the stars were a little closer together, it would be practically impossible to separate them.

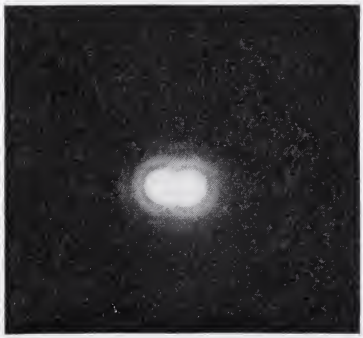


FIG. 93

Fig. 94 represents the case of a triple star whose components are so close together as to be barely within the limit of resolution of the telescope. In this case the object would probably be taken as triple because its central portion is triangular. If the three stars were a little closer together, it would be impossible to say whether the object viewed were a single or a double star, or a triple star, or a circular disc.

FIG. 94

If now, in measuring the distance between two double stars, or the diameter of a disc such as that presented by a small satellite or one of the minor planets, instead of attempting to measure what is usually called the "edge" of the disc — which, as before stated, is a very uncertain thing and varies with the observer and with atmospheric conditions — we try to find a relation between the size and shape of the object and the clearness of

the interference fringes, we should have a means of making an independent measurement of the size of objects which are practically beyond the power of resolution of the most powerful telescope. The principal object of this lecture is to show the feasibility of such methods of measurement. For this purpose, however, the circular fringes that we have been investigating are not very well adapted; they are not very sharply defined; there is not enough contrast between them. However, there is a relation which can be traced out between the clearness of the diffraction fringes and the size and shape of the object viewed. This relation is very complex.

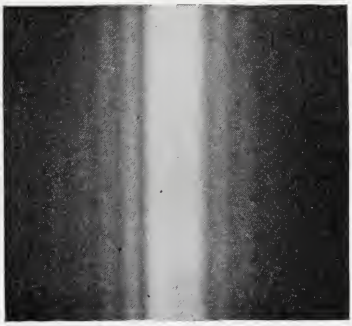


FIG. 95

The result of such calculation is that the intensity is greatest at the center, whence it rapidly falls off to zero at the first dark band. It then increases to a second maximum, where it is not more than one-ninth as great as in the center. What we should have to observe, then, is the contrast between these two parts—one but one-ninth as marked as the other and confused more or less by atmospheric disturbances. In case of a rectangular aperture the intensity curve is somewhat different, in that the maxima on either side of the central band are considerably greater, so that it is somewhat easier to see the fringes. In case of the rectangular aperture the fringes are parallel to the long sides of the rectangle. The appearance of the diffraction phenomenon in this case is illustrated in Fig. 95. The pattern consists of a broad central space, whose sides are parallel to the sides of the rectangular slit, and of a succession of fringes diminishing in intensity on

either side. The corresponding intensity curve is shown in Fig. 96.[1]

If we had two such apertures instead of one, the appearance would be all the more definite; but the two apertures together produce, in addition, interference fringes very much finer than the others, but very sharp and clear. The intensity curve corresponding to these two slits is shown in Fig. 97. In this case it is easy to distinguish the successive maxima, and the atmospheric disturbances are very much less harmful than in the case of the more indefinite phenomenon.

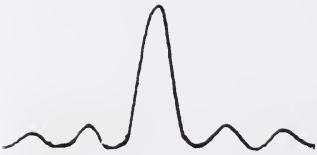


FIG. 96

Fig. 98 represents the appearance of the diffraction pattern due to two slits when a slit, instead of a point, is used as the source of light. The appearance of the two patterns is not essentially different, that due to the slit being very much brighter. In the case of a point source there is so little light that it is more difficult to see the fringes. Here the same large fringes are visible as before, but over the central bright space there is a number of very fine fringes. The two central ones are particularly sharp, so that it is easy to locate their position if necessary, but still easier to determine their visibility. This clearness is the essential point we have to consider, because the size of the object determines the clearness of the fringes. We find that if we gradually increase the width of the source, the fringes grow less and less distinct, and finally disappear entirely. If we note the instant when the fringes disappear, we can calculate from the dimensions of the apparatus the width of the

FIG. 97

[1] This ignores the diffraction bands parallel to the shorter sides of the rectangle, which are usually inconspicuous.

source. Or, if we alter the dimensions of the apparatus and observe when the fringes cease to be visible in our observing telescope, we have the means of measuring the diameter of the source, which may be a double star, or the disc of one of Jupiter's satellites, or one of the minor planets.

We may get some notion of the relation which exists between the clearness of the fringes and the size of the object when the fringes disappear, by considering a simple case like that of a double star. Suppose we have two slits in front of the object glass of a telescope focused on a single star. At the focus the rays from the two slits come together in condition to produce interference fringes, and the fringes always appear when the source is a point. Suppose we have in the field of view another star. It will produce its own series of fringes in the focus of the telescope. We shall then have two similar sets of fringes in the field of view. If, now, the two stars are so near together that the central bright fringes of the two systems coincide, then the two sets of fringes will reinforce each other. If, however, one of the stars is just so far away from the other that the angle between them is equal to the angle between the central bright band and its first adjacent minimum, then the maximum of one system of fringes will fall upon the minimum of the other set, and the two will efface each other so that the fringes disappear. Hence the fringes disappear when the angle subtended by the source is equal to the angle subtended by half the breadth of the fringes, viewed from the objec-

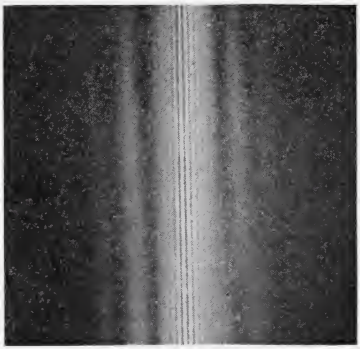


FIG. 98

tive. This angle is easily calculated. Thus if l represent the wave length and s is the distance between the two slits, then the angle is equal to $\frac{1}{2} \cdot \frac{l}{s}$. Hence, if we know the length of the light wave (we can take it as one fifty-thousandth of an inch if we choose), by measuring the distance between our slits when the fringes disappear we have the means of measuring the angular distance between double stars.

In the case of a single-slit source we can also get some sort of an idea of the conditions which prevail when the fringes disappear. For we may conceive the slit source to be divided into a number of line sources, parallel and adjacent to each other. Then each line source would form its own set of fringes, and when the angle between the two outside lines, *i. e.*, the edges of the slit, is equal to the angle subtended by the distance of the first dark band from the center, the fringes again overlap in such a way as to disappear. The value of this angle is easily found to be $\frac{l}{s}$. So, supposing that we had such an object in the heavens as a narrow band of light, we have the means of finding its width. If, instead of a slit, we used a circular opening as a source, there is a little more difficulty in the mathematical analysis. In this case the coefficient of $\frac{l}{s}$, instead of being 1 as in the second case, or $\frac{1}{2}$ as in the first case, is found to be 1.22. In observing such an object we measure the distance between our two slits when the interference fringes have just vanished, and compute the angular magnitude of the object by using this coefficient. If we knew the distance to the object, we could calculate also its actual diameter.

The curve representing the clearness of the fringes as the slits approach is rather interesting. It varies with the form

of the object viewed. In the case of a double star it falls very rapidly from its maximum to zero; then it rises again, and if the two slits themselves could possibly be infinitely narrow and the light perfectly homogeneous, it would rise to its original value. But because the slits themselves have a certain width, and because the observation is usually made with white light, this second maximum is usually less than the first.

If the source is a single point of light, then the fringes are equally distinct, no matter what the distance between the slits; whereas, when the source is a disc of appreciable angular width, the fringes fade out as the distance between the slits increases, so that there is no possibility of a doubt as to whether we are looking at a point or a source of appreciable size.

Suppose we are looking at a disc of a given diameter through such a pair of slits which are close together. If we gradually increase the distance between the slits, the visibility becomes smaller and smaller until the fringes disappear entirely. As the distance between the slits increases again, the clearness increases, and so on; *i. e.*, there are subsequent maxima and minima which may be measured, if it be considered desirable. It is necessary, however, to measure this distance between the two slits at the time the fringes first disappear; we may measure this distance at the subsequent disappearances if we choose, but it is not essential, for we are able to find the diameter of the object (the distance between two objects in the case of the double star) if we know the distance between the slits at the first disappearance. If, however, we do not know the shape of the source, we must observe at least one more disappearance.

In Fig. 99 the visibility curves which characterize a slit, a uniformly illuminated disc, and a disc whose intensity is greater at the center, are shown. The full curve corresponds to a slit, the dotted one to a disc, and the dashed

one to the disc which is brighter at the center. It will be noted that in the case of the slit the distances between the zero points are all alike. In the case of the disc the curve is still of the same general form, but the distance to the first zero position is no longer equal to the others, but is 1.22 as great. Hence, if the distances between the zero points are equal, as shown in the figure for the full curve, we know the

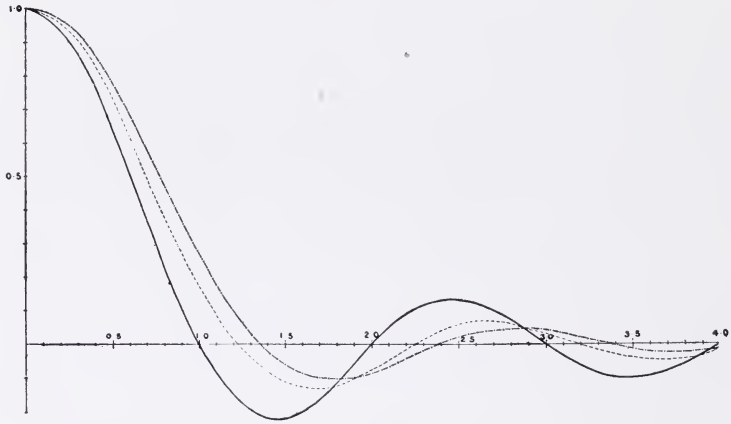


FIG. 99

source is rectangular. But if the distance to the first zero point is 1.22 times as great as the distances between the succeeding zero points, we know that we are observing a uniformly illuminated circular object. The next interval would determine in this case, as in the first, the diameter of the object viewed.

In the case of the slit the distances between the zero points are rigorously equal, and it may be of interest to note that the visibility at the second maximum is something like one-fourth of the visibility at the first. So there is no possibility of deception in noting the point at which the fringes disappear; indeed, the disappearance can be so sharply determined that we may measure the corresponding distance be-

tween the slits to within 1 per cent. of its whole value, and so determine the width of the line source with a corresponding degree of accuracy.

The visibility curve shown in Fig. 100 represents the case in which the source is a double disc — a double star, for instance, in which the discs have appreciable magnitude. The envelope of the curve, which is drawn full, corresponds to the circular form of the separate discs, and from this curve we can determine the size of the separate discs, provided they are equal. The dotted curve tells us that we are dealing with a double object. Hence, if in observing a heavenly body we obtain a visibility curve of this form, we infer that we are dealing with a double star.

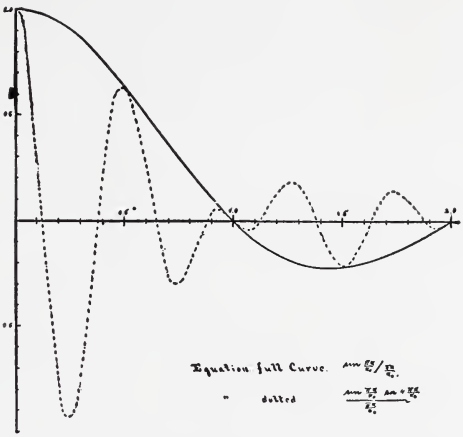


FIG. 100

There is a difficulty in carrying out such observations, especially when we are observing a very small object or a very close double star. For in this case the slits have to be separated rather widely, and the angle between the rays from the two slits, when they come together, is rather large. Hence, the distance between the interference fringes is correspondingly small, as was shown in a previous lecture, and this distance becomes less and less as the angle becomes greater and greater. When we approach the limit of resolution of the telescope, the fringes are so small that a rather high power eyepiece must be used in order to see

them, and the light is correspondingly feeble. We may overcome this difficulty in the same way as we did in our transformation of the microscope into the interferometer, by using mirrors to change the direction of the beam of light, instead of allowing it to pass through two apertures in front of the lens.

Fig. 101 represents two arrangements by which this may be accomplished. The light falls from above upon the two

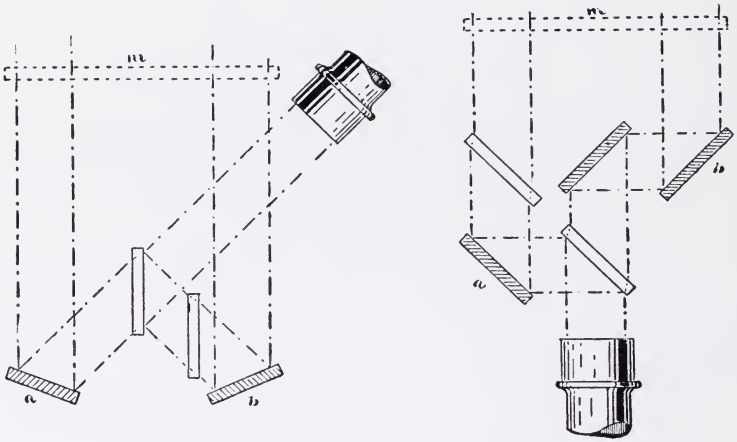


FIG. 101

mirrors a and b, which correspond to the two slits. By these mirrors we can bend the light at any angle we choose, and bring the two beams together again at as small an angle as we wish, by means of the plane-parallel plate. Thus we can make the fringes as broad as we choose. In the second diagram we have a rather more complex arrangement of mirrors, but the effect is the same. The paths of the two rays can be easily traced in the diagrams.

If we wish to observe with such an arrangement a body of the size of a small satellite, we should have to construct the instrument so that the distance between the two mirrors could be altered, because these mirrors correspond to the

two slits whose distance apart must be changed. This can be done by mounting the mirror *a* and the mirror *b* on a right- and left-handed screw. On turning the screw the two mirrors would move in opposite directions through equal distances, leaving everything else unchanged. Such an instrument is represented in Fig. 102. The light falls from below upon the two mirrors *a* and *b*, which are mounted on carriages which can be moved in opposite directions by the right- and left-handed screw.

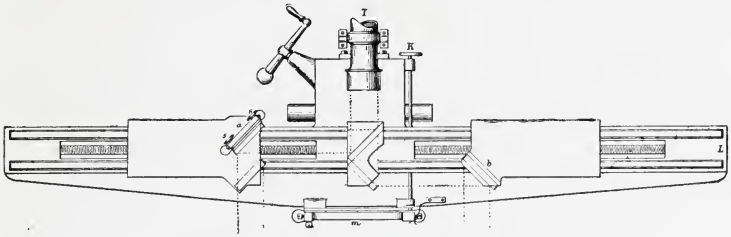


FIG. 102

Fig. 103 represents an actual instrument which was used in making laboratory experiments to test the method. The artificial double stars, or star discs, were pinholes made in a sheet of platinum. These holes were as small as it was possible to make them, of such a diameter as to test the resolution of the telescope, with a bright source of light behind them. The left-hand figure represents the double slit. It is mounted on a right- and left-handed screw and can be operated by the observer. The slits can thus be moved by a measurable quantity, and their distance apart when the fringes disappear can be determined.

After making a series of such experiments in the laboratory, I was invited to spend a few weeks at the Lick Observatory at Mount Hamilton to test the method on Jupiter's satellites. These satellites have angular magnitudes of something like one second of arc, so that they should be measurable by this

method. The actual micrometric measurements which have been made of these satellites with the largest telescopes give results which vary considerably among themselves. Hence the interest in trying the interferometer method. The apparatus used was similar to that shown in Fig. 103, *i. e.*, it consisted of two movable slits in front of the objective of the eleven-inch glass at the Lick Observatory.

The atmospheric conditions at Mount Hamilton while the work was in progress were not altogether favorable, so that

FIG. 103

out of the three weeks' sojourn there there were only four nights which were good enough to use, though one of these nights was almost perfect; and on this one night most of the measurements were made. The results obtained, together with those of four determinations which have been made by the ordinary micrometer method, using the largest telescopes available, are given in the following table:

Number of Satellite	A. A. M.	Eng.	St.	Ho.	Bu.
I	1.02	1.08	1.02	1.11	1.11
II	0.94	0.91	0.91	0.98	1.00
III	1.37	1.54	1.49	1.78	1.78
IV	1.31	1.28	1.27	1.46	1.61

The numbers in the column marked A. A. M. are the results in seconds of arc obtained by the interference method. The other columns contain the results obtained by the ordinary method by Engelmann, Struve, Hough, and Burnham

respectively. The important point to be noted is that the results by the interference method are near the mean of the other results, and that the results obtained by the other method differ widely among themselves.

It is also important to note that, while an eleven-inch glass was used for the observations by the interference method, the distance between the slits at which the fringes disappear was very much less than eleven inches; on the average, something like four inches. Now, with a six-inch glass one can easily put two slits at a distance of four inches. Hence a six-inch glass can be used with the same effectiveness as the eleven-inch, and gives results by the interference method which are equal in accuracy to those obtained by the largest telescopes known. If this same method were applied to the forty-inch glass of the Yerkes Observatory, it would certainly be possible to obtain measurements of objects only one-sixth as large as the satellites of Jupiter.

The principal object of the method which has been described was not, however, to measure the diameter of the planets and satellites, or even of the double stars, though it seems likely now that this will be one rather important object that may be accomplished by it; for some double stars are so close together that it is impossible to separate them in the largest telescope. A more ambitious problem, which may not be entirely hopeless, is that of measuring the diameter of the stars themselves. The nearest of these stars, as before stated, is so far away that it takes several years for light from it to reach us. They are about 100,000 times as far away as the sun. If they were as large as the sun, the angle they would subtend would be about one-hundredth of a second. A forty-inch telescope can resolve angles of approximately one-tenth of a second, so that, if we were to attempt to measure, or to observe, a disc of only

one-hundredth of a second, it would require an objective whose diameter is of the order of forty feet — which, of course, is out of the question. It is, however, not altogether out of the question to construct an interference apparatus such that the distance between its mirrors would be of this order of magnitude.

But it is not altogether improbable that even some of the nearer stars are considerably larger than the sun, and in that case the angle which they subtend would be considerably larger. Hence it might not be necessary to have an instrument with mirrors forty feet apart. In addition it may be noted that it is not absolutely necessary to observe the disappearance of the fringes in order to show that the object has definite magnitude; for if the visibility of the fringes varies at all, we know that the source is not a point. For, suppose we observe the visibility curve of a star which is so far away that we know it has no appreciable disc. The visibility curve would correspond to a straight line. There would be no appreciable difference in distinction of fringes as the distance between the slits was increased indefinitely. If we now observe a star which has a diameter of one-hundredth of a second, we need only to observe that the visibility for a large distance between the slits is less than in the case of the distant star, in order to know that the second object has an appreciable disc, even if the instruments were not large enough to increase the distance sufficiently to make the fringes disappear. From the difference between two such visibility curves we might calculate rather roughly the actual magnitude of the stars.

SUMMARY

1. The investigation of the size and structure of the heavenly bodies is limited by the resolving power of the observing telescope. When the bodies are so small or so

distant that this limit of resolution is passed, the telescope can give no information concerning them.

2. But an observation of the visibility curves of the interference fringes due to such sources, when made by the method of the double slit or its equivalent, and properly interpreted, gives information concerning the size, shape, and distribution of the components of the system. Even in the case of a fixed star, which may subtend an angle of less than one-hundredth of a second, it may not be an entirely hopeless task to attempt to measure its diameter by this means.

LECTURE VIII

THE ETHER

THE velocity of light is so enormously greater than anything with which we are accustomed to deal that the mind has some little difficulty in grasping it. A bullet travels at the rate of approximately half a mile a second. Sound, in a steel wire, travels at the rate of three miles a second. From this — if we agree to except the velocities of the heavenly bodies — there is no intermediate step to the velocity of light, which is about 186,000 miles a second. We can, perhaps, give a better idea of this velocity by saying that light will travel around the world seven times between two ticks of a clock.

Now, the velocity of wave propagation can be seen, without the aid of any mathematical analysis, to depend on the elasticity of the medium and its density; for we can see that if a medium is highly elastic the disturbance would be propagated at a great speed. Also, if the medium is dense the propagation would be slower than if it were rare. It can easily be shown that if the elasticity were represented by E, and the density by D, the velocity would be represented by the square root of E divided by D. So that, if the density of the medium which propagates light waves were as great as the density of steel, the elasticity, since the velocity of light is some 60,000 times as great as that of the propagation of sound in a steel wire, must be 60,000 squared times as great as the elasticity of steel. Thus, this medium which propagates light vibrations would have to have an elasticity of the order of 3,600,000,000 times the elasticity of steel. Or, if the elasticity of the medium were the same

as that of steel, the density would have to be 3,600,000,000 times as small as that of steel, that is to say, roughly speaking, about 50,000 times as small as the density of hydrogen, the lightest known gas. Evidently, then, a medium which propagates vibrations with such an enormous velocity must have an enormously high elasticity or abnormally low density. In any case, its properties would be of an entirely different order from the properties of the substances with which we are accustomed to deal, so that it belongs in a category by itself.

Another course of reasoning which leads to this same conclusion — namely, that this medium is not any ordinary form of matter, such as air or gas or steel — is the following: Sound is produced by a bell under a receiver of an air pump. When the air has the same density inside the receiver as outside, the sound reaches the ear of an observer without difficulty. But when the air is gradually pumped out of the receiver, the sound becomes fainter and fainter until it ceases entirely. If the same thing were true of light, and we exhausted a vessel in which a source of light — an incandescent lamp, for example—had been placed, then, after a certain degree of exhaustion was reached, we ought to see the light less clearly than before. We know, however, that the contrary is the case, *i. e.*, that the light is actually brighter and clearer when the exhaustion of the receiver has been carried to the highest possible degree. The probabilities are enormously against the conclusion that light is transmitted by the very small quantity of residual gas. There are other theoretical reasons, into which we will not enter.

Whatever the process of reasoning, we are led to the same result. We know that light vibrations are transverse to the direction of propagation, while sound vibrations are in the direction of propagation. We know also that in the case of a solid body transverse vibrations can be readily trans-

mitted. Thus, if we have a long cylindrical rod and we give one end of it a twist, the twist will travel along from one end to the other. If the medium, instead of being a solid rod, were a tube of liquid, and were twisted at one end, there would be no corresponding transmission of the twist to the other end, for a liquid cannot transmit a torsional strain. Hence this reasoning leads to the conclusion that if the medium which propagates light vibrations has the properties of ordinary matter, it must be considered to be an elastic solid rather than a fluid.

This conclusion was considered one of the most formidable objections to the undulatory theory that light consists of waves. For this medium, notwithstanding the necessity for the assumption that it has the properties of a solid, must yet be of such a nature as to offer little resistance to the motion of a body through it. Take, for example, the motion of the planets around the sun. The resistance of the medium is so small that the earth has been traveling around the sun millions of years without any appreciable increase in the length of the year. Even the vastly lighter and more attenuated comets return to the same point periodically, and the time of such periodical returns has been carefully noted from the earliest historical times, and yet no appreciable increase in it has been detected. We are thus confronted with the apparent inconsistency of a solid body which must at the same time possess in such a marked degree the properties of a perfect fluid as to offer no appreciable resistance to the motion of bodies so very light and extended as the comets. We are, however, not without analogies, for, as was stated in the first lecture, substances such as shoemaker's wax show the properties of an elastic solid when reacting against rapid motions, but act like a liquid under pressures.

In the case of shoemaker's wax both of these contradictory

properties are very imperfectly realized, but we can argue from this fact that the medium which we are considering might have the various properties which it must possess in an enormously exaggerated degree. It is, at any rate, not at all inconceivable that such a medium should at the same time possess both properties. We know that the air itself does not possess such properties, and that no matter which we know possesses them in sufficient degree to account for the propagation of light. Hence the conclusion that light vibrations are not propagated by ordinary matter, but by something else. Cogent as these three lines of reasoning may be, it is undoubtedly true that they do not always carry conviction. There is, so far as I am aware, no process of reasoning upon this subject which leads to a result which is free from objection and absolutely conclusive.

But these are not the only paradoxes connected with the medium which transmits light. There was an observation made by Bradley a great many years ago, for quite another purpose. He found that when we observe the position of a star by means of the telescope, the star seems shifted from its actual position, by a certain small angle called the angle of aberration. He attributed this effect to the motion of the earth in its orbit, and gave an explanation of the phenomenon which is based on the corpuscular theory and is apparently very simple. We will give this explanation, notwithstanding the fact that we know the corpuscular theory to be erroneous.

Let us suppose a raindrop to be falling vertically and an observer to be carrying, say, a gun, the barrel being as nearly vertical as he can hold it. If the observer is not moving and the raindrop falls in the center of the upper end of the barrel, it will fall centrally through the lower end. Suppose, however, that the observer is in motion

in the direction *bd* (Fig. 104); the raindrop will still fall exactly vertically, but if the gun advances laterally while the raindrop is within the barrel, it strikes against the side. In order to make the raindrop move centrally along the axis of the barrel, it is evidently necessary to incline the gun at an angle such as *bad*. The gun barrel is now pointing, apparently, in the wrong direction, by an angle whose tangent is the ratio of the velocity of the observer to the velocity of the raindrop.

FIG. 104

According to the undulatory theory, the explanation is a trifle more complex; but it can easily be seen that, if the medium we are considering is motionless and the gun barrel represents a telescope, and the waves from the star are moving in the direction *ad*, they will be concentrated at a point which is in the axis of the telescope, unless the latter is in motion. But if the earth carrying the telescope is moving with a velocity something like twenty miles a second, and we are observing the stars in a direction approximately at right angles to the direction of that motion, the light from the star will not come to a focus on the axis of the telescope, but will form an image in a new position, so that the telescope appears to be pointing in the wrong direction. In order to bring the image on the axis of the instrument, we must turn the telescope from its position through an angle whose tangent is the ratio of the velocity of the earth in its orbit to the velocity of light. The velocity of light is, as before stated, 186,000 miles a second—200,000 in round numbers—and the velocity of the earth in its orbit is roughly twenty miles a second. Hence the tangent of the angle of aberration would be measured by the ratio of 1 to 10,000.

More accurately, this angle is 20″.445. The limit of accuracy of the telescope, as was pointed out in several of the preceding lectures, is about one-tenth of a second; but, by repeating these measurements under a great many variations in the conditions of the problem, this limit may be passed, and it is practically certain that this number is correct to the second decimal place.

When this variation in the apparent position of the stars was discovered, it was accounted for correctly by the assumption that light travels with a finite velocity, and that, by measuring the angle of aberration, and knowing the speed of the earth in its orbit, the velocity of light could be found. This velocity has since been determined much more accurately by experimental means, so that now we use the velocity of light to deduce the velocity of the earth and the radius of its orbit.

The objection to this explanation was, however, raised that if this angle were the ratio of the velocity of the earth in its orbit to the velocity of light, and if we filled a telescope with water, in which the velocity of light is known to be only three-fourths of what it is in air, it would take one and one-third times as long for the light to pass from the center of the objective to the cross-wires, and hence we ought to observe, not the actual angle of aberration, but one which should be one-third greater. The experiment was actually tried. A telescope was filled with water, and observations on various stars were continued throughout the greater part of the year, with the result that almost exactly the same value was found for the angle of aberration.

This result was considered a very serious objection to the undulatory theory until an explanation was found by Fresnel. He proposed that we consider that the medium which transmits the light vibrations is carried along by the motion of the water in the telescope in the direction of the motion of the

earth around the sun. Now, if the light waves were carried along with the full velocity of the earth in its orbit, we should be in the same difficulty, or in a more serious difficulty, than before. Fresnel, however, made the further supposition that the velocity of the carrying along of the light waves by the motion of the medium was less than the actual velocity of the medium itself, by a quantity which depended on the index of refraction of the substance. In the case of water the value of this factor is seven-sixteenths.

This, at first sight, seems a rather forced explanation; indeed, at the time it was proposed it was treated with considerable incredulity. An experiment was made by Fizeau, however, to test the point — in my opinion one of the most ingenious experiments that have ever been attempted in the whole domain of physics. The problem is to find the increase in the velocity of light due to a motion of the medium. We have an analogous problem in the case of sound, but in this case it is a very much simpler matter. We know by actual experiment, as we should infer without experiment, that the velocity of sound is increased by the velocity of a wind which carries the air in the same direction, or diminished if the wind moves in the opposite direction. But in the case of light waves the velocity is so enormously great that it would seem, at first sight, altogether out of the question to compare it with any velocity which we might be able to obtain in a transparent medium such as water or glass. The problem consists in finding the change in the velocity of light produced by the greatest velocity we can get — about twenty feet a second — in a column of water through which light waves pass. We thus have to find a difference of the order of twenty feet in 186,000 miles, *i. e.*, of one part in 50,000,000. Besides, we can get only a relatively small column of water to pass light through and still see the light when it returns.

The difficulty is met, however, by taking advantage of the excessive minuteness of light waves themselves. This double length of the water column is something like forty feet. In this forty feet there are, in round numbers, 14,000,000 waves. Hence the difference due to a velocity of twenty feet per second, which is the velocity of the water current, would produce a displacement of the interference fringes (produced by two beams, one of which passes down the column and the other up the column of the moving liquid) of about one-half a fringe, which corresponds to a difference of one-half a light wave in the paths. Reversing the water current should produce a shifting of one-half a fringe in the opposite direction, so that the total shifting would actually be of the order of one interference fringe. But we can easily observe one-tenth of a fringe, or in some cases even less than that. Now, one fringe would be the displacement if water is the medium which transmits the light waves. But this other medium we have been talking about moves, according to Fresnel, with a smaller velocity than the water, and the ratio of the velocity of the medium to the velocity of the water should be a particular fraction, namely, seven-sixteenths. In other words, then, instead of the whole fringe we ought to get a displacement of seven-sixteenths of a fringe by the reversal of the water current. The experiment was actually tried by Fizeau, and the result was that the fringes were shifted by a quantity less than they should have been if water had been the medium; and hence we conclude that the water was not the medium which carried the vibrations.

The arrangement of the apparatus which was used in the experiment is shown in Fig. 105. The light starts from a narrow slit S, is rendered parallel by a lens L, and separated into two pencils by apertures in front of the two tubes TT, which carry the column of water. Both tubes are closed by

pieces of the same plane-parallel plate of glass. The light passes through these two tubes and is brought to a focus by the lens in condition to produce interference fringes. The apparatus might have been arranged in this way but for the fact that there would be changes in the position of the interference fringes whenever the density or temperature of the medium changed; and, in particular, whenever the current changes direction there would be produced alterations in length and changes in density; and these exceedingly

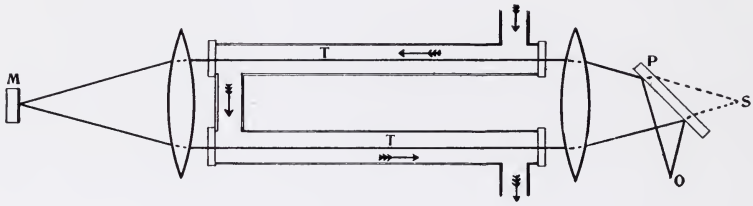


FIG. 105

slight differences are quite sufficient to account for any motion of the fringes. In order to avoid this disturbance, Fresnel had the idea of placing at the focus of the lens the mirror M, so that the two rays return, the one which came through the upper tube going back through the lower, and *vice versa* for the other ray. In this way the two rays pass through identical paths and come together at the same point from which they started. With this arrangement, if there is any shifting of the fringes, it must be due to the reversal of the change in velocity due to the current of water. For one of the two beams, say the upper one, travels with the current in both tubes; the other, starting at the same point, travels against the current in both tubes. Upon reversing the direction of the current of water the circumstances are exactly the reverse: the beam which before traveled with the current now travels against it, etc. The result of the experiment, as before stated, was that there was produced a

displacement of less than should have been produced by the motion of the liquid. How much less was not determined. To this extent the experiment was imperfect.

On this account, and also for the reason that the experiment was regarded as one of the most important in the entire subject of optics, it seemed to me that it was desirable to repeat it

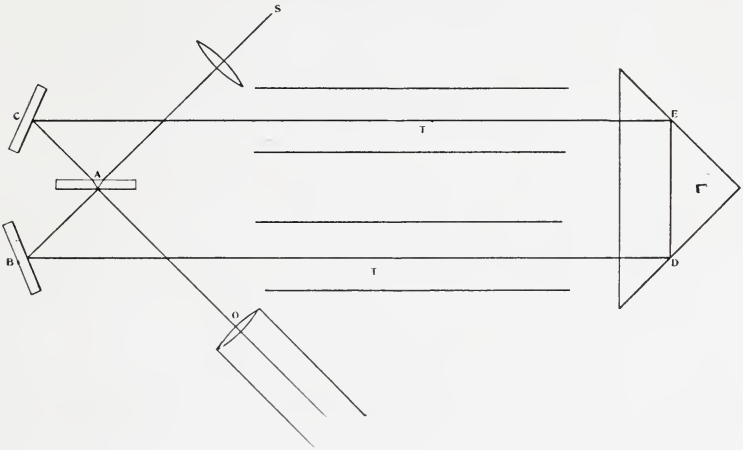


FIG. 106

in order to determine, not only the fact that the displacement was less than could be accounted for by the motion of the water, but also, if possible, how much less. For this purpose the apparatus was modified in several important points, and is shown in Fig. 106.

It will be noted that the principle of the interferometer has been used to produce interference fringes of considerable breadth without at the same time reducing the intensity of the light. Otherwise, the experiment is essentially the same as that made by Fizeau. The light starts from a bright flame of ordinary gas light, is rendered parallel by the lens, and then falls on the surface, which divides it into two parts, one reflected and one transmitted. The reflected

portion goes down one tube, is reflected twice by the total reflection prism P through the other tube, and passes, after necessary reflection, into the observing telescope. The other ray pursues the contrary path, and we see interference fringes in the telescope as before, but enormously brighter and more definite. This arrangement made it possible to make measurements of the displacement of the fringes which were very accurate. The result of the experiment was that the measured displacement was almost exactly seven-sixteenths of what it would have been had the medium which transmits the light waves moved with the velocity of the water.

It was at one time proposed to test this problem by utilizing the velocity of the earth in its orbit. Since this velocity is so very much greater than anything we can produce at the earth's surface, it was supposed that such measurements could be made with considerable ease; and they were actually tried in quite a considerable number of different ways and by very eminent men. The fact is, we cannot utilize the velocity of the earth in its orbit for such experiments, for the reason that we have to determine our directions by points outside of the earth, and the only thing we have is the stars, and the stars are displaced by this very element which we want to measure; so the results would be entirely negative. It was pointed out by Lorentz that it is impossible by any measurements made on the surface of the earth to detect any effect of the earth's motion.

Maxwell considered it possible, theoretically at least, to deal with the square of the ratio of the two velocities; that is, the square of $\frac{1}{10000}$, or $\frac{1}{100000000}$. He further indicated that if we made two measurements of the velocity of light, one in the direction in which the earth is travel ing in its orbit, and one in a direction at right angles to this, then the time it takes light to pass over the same

length of path is greater in the first case than in the second.

We can easily appreciate the fact that the time is greater in this case, by considering a man rowing in a boat, first in a smooth pond and then in a river. If he rows at the rate of four miles an hour, for example, and the distance between the stations is twelve miles, then it would take him three hours to pull there and three to pull back — six hours in all. This is his time when there is no current. If there is a current, suppose at the rate of one mile an hour, then the time it would take to go from one point to the other, would be, not 12 divided by 4, but 12 divided by $4 + 1$, *i. e.*, 2.4 hours. In coming back the time would be 12 divided by $4 - 1$, which would be 4 hours, and this added to the other time equals 6.4 instead of 6 hours. It takes him longer, then, to pass back and forth when the medium is in motion than when the medium is at rest. We can understand, then, that it would take light longer to travel back and forth in the direction of the motion of the earth. The difference in the times is, however, so exceedingly small, being of the order of 1 in 100,000,000, that Maxwell considered it practically hopeless to attempt to detect it.

In spite of this apparently hopeless smallness of the quantities to be observed, it was thought that the minuteness of the light waves might again come to our rescue. As a matter of fact, an experiment was devised for detecting this small quantity. The conditions which the apparatus must fulfil are rather complex. The total distance traveled must be as great as possible, something of the order of one hundred million waves, for example. Another condition requires that we be able to interchange the direction without altering the adjustment by even the one hundredth-millionth part. Further, the apparatus must be absolutely free from vibration.

The problem was practically solved by reflecting part of the light back and forth a number of times and then returning it to its starting-point. The other path was at right angles to the first, and over it the light made a similar series of excursions, and was also reflected back to the starting-point. This starting-point was a separating plane in an interferometer, and the two paths at right angles were the two arms of an interferometer. Notwithstanding the very considerable difference in path, which must involve an exceedingly high order of accuracy in the reflecting surfaces and a constancy of temperature in the air between, it was possible to see fringes and to keep them in position for several hours at a time.

These conditions having been fulfilled, the apparatus was mounted on a stone support, about four feet square and one foot thick, and this stone was mounted on a circular disc of wood which floated in a tank of mercury. The resistance to motion is thus exceedingly small, so that by a very slight pressure on the circumference the whole could be kept in slow and continuous rotation. It would take, perhaps, five minutes to make one single turn. With this slight motion there is practically no oscillation; the observer has to follow around and at intervals to observe whether there is any displacement of the fringes.

It was found that there was no displacement of the interference fringes, so that the result of the experiment was negative and would, therefore, show that there is still a difficulty in the theory itself; and this difficulty, I may say, has not yet been satisfactorily explained. I am presenting the case, not so much for solution, but as an illustration of the applicability of light waves to new problems.

The actual arrangement of the experiment is shown in Fig. 107. A lens makes the rays nearly parallel. The dividing surface and the two paths are easily recognized. The telescope was furnished with a micrometer screw to determine

the amount of displacement of the fringes, if there were any. The last mirror is mounted on a slide; so these two paths may be made equal to the necessary degree of accuracy—something of the order of one fifty-thousandth of an inch.

Fig. 108 represents the actual apparatus. The stone and the circular disc of wood supporting the stone in the tank filled with mercury are readily recognized; also the dividing surface and the various mirrors.

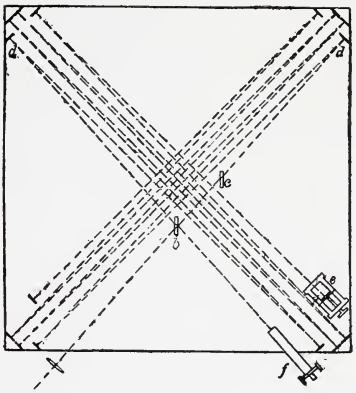


FIG. 107

It was considered that, if this experiment gave a positive result, it would determine the velocity, not merely of the earth in its orbit, but of the earth through the ether. With good reason it is supposed that the sun and all the planets as well are moving through space at a rate of perhaps twenty miles per second in a certain particular direction. The velocity is not very well determined, and it was hoped that with this experiment we could measure this velocity of the whole solar system through space. Since the result of the experiment was negative, this problem is still demanding a solution.

The experiment is to me historically interesting, because it was for the solution of this problem that the interferometer was devised. I think it will be admitted that the problem, by leading to the invention of the interferometer, more than compensated for the fact that this particular experiment gave a negative result.

From all that precedes it appears practically certain that there must be a medium whose proper function it is to transmit light waves. Such a medium is also necessary for the

transmission of electrical and magnetic effects. Indeed, it is fairly well established that light is an electro-magnetic disturbance, like that due to a discharge from an induction coil or a condenser. Such electric waves can be reflected and refracted and polarized, and be made to produce vibrations

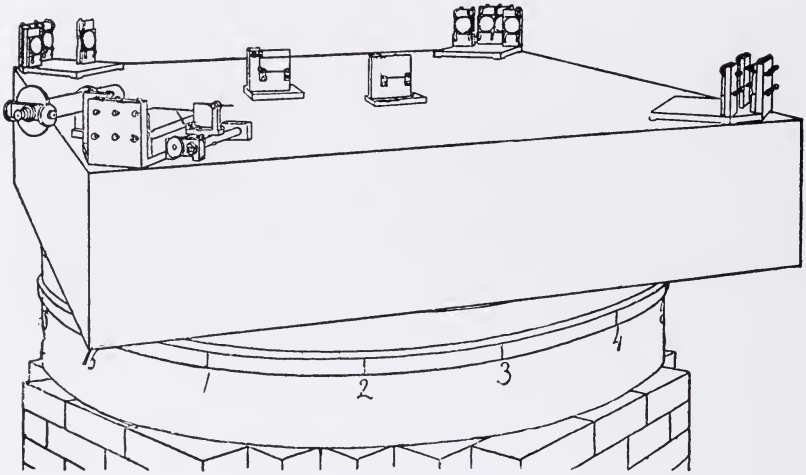


FIG. 108

and other changes, just as the light waves can. The only difference between them and the light waves is in the wave length.

This difference may be enormous or quite moderate. For example, a telegraphic wave, which is practically an electro-magnetic disturbance, may be as long as one thousand miles. The waves produced by the oscillations of a condenser, like a Leyden jar, may be as short as one hundred feet; the waves produced by a Hertz oscillator may be as short as one-tenth of an inch. Between this and the longest light wave there is not an enormous gap, for the latter has a length of about one-thousandth of an inch. Thus the difference between the

Hertz vibrations and the longest light wave is less than the difference between the longest and shortest light waves, for some of the shortest oscillations are only a few millionths of an inch long. Doubtless even this gap will soon be bridged over.

The settlement of the fact that light is a magneto-electric oscillation is in no sense an explanation of the nature of light. It is only a transference of the problem, for the question then arises as to the nature of the medium and of the mechanical actions involved in such a medium which sustains and transmits these electro-magnetic disturbances.

A suggestion which is very attractive on account of its simplicity is that the ether itself is electricity; a much more probable one is that electricity is an ether strain — that a displacement of the ether is equivalent to an electric current. If this is true, we are returning to our elastic-solid theory. I may quote a statement which Lord Kelvin made in reply to a rather skeptical question as to the existence of a medium about which so very little is supposed to be known. The reply was: "Yes, ether is the only form of matter about which we know anything at all." In fact, the moment we begin to inquire into the nature of the ultimate particles of ordinary matter, we are at once enveloped in a sea of conjecture and hypotheses—all of great difficulty and complexity.

One of the most promising of these hypotheses is the "ether vortex theory," which, if true, has the merit of introducing nothing new into the hypotheses already made, but only of specifying the particular form of motion required. The most natural form of such vortex motions with which to deal is that illustrated by ordinary smoke rings, such as are frequently blown from the stack of a locomotive. Such vortex rings may easily be produced by filling with smoke a box which has a circular aperture at one end and a rubber diaphragm at the other, and then tapping the rubber. The

friction against the side of the opening, as the puff of smoke passes out, produces a rotary motion, and the result will be smoke rings or vortices.

Investigation shows that these smoke rings possess, to a certain degree, the properties which we are accustomed to associate with atoms, notwithstanding the fact that the medium in which these smoke rings exists is far from ideal. If the medium were ideal, it would be devoid of friction, and then the motion, when once started, would continue indefinitely, and that part of the ether which is differentiated by this motion would ever remain so.

Another peculiarity of the ring is that it cannot be cut —it simply winds around the knife. Of course, in a very short time the motion in a smoke ring ceases in consequence of the viscosity of the air, but it would continue indefinitely in such a frictionless medium as we suppose the ether to be.

There are a number of other analogies which we have not time to enter into — quite a number of details and instances of the interactions of the various atoms which have been investigated. In fact, there are so many analogies that we are tempted to think that the vortex ring is in reality an enlarged image of the atom. The mathematics of the subject is unfortunately very difficult, and this seems to be one of the principal reasons for the slow progress made in the theory.

Suppose that an ether strain corresponds to an electric charge, an ether displacement to the electric current, these ether vortices to the atoms — if we continue these suppositions, we arrive at what may be one of the grandest generalizations of modern science — of which we are tempted to say that it ought to be true even if it is not — namely, that all the phenomena of the physical universe are only different manifestations of the various modes of motions of one all-pervading substance — the ether.

All modern investigation tends toward the elucidation of this problem, and the day seems not far distant when the converging lines from many apparently remote regions of thought will meet on this common ground. Then the nature of the atoms, and the forces called into play in their chemical union; the interactions between these atoms and the non-differentiated ether as manifested in the phenomena of light and electricity; the structures of the molecules and molecular systems of which the atoms are the units; the explanation of cohesion, elasticity, and gravitation — all these will be marshaled into a single compact and consistent body of scientific knowledge.

SUMMARY

1. A number of independent courses of reasoning lead to the conclusion that the medium which propagates light waves is not an ordinary form of matter. Little as we know about it, we may say that our ignorance of ordinary matter is still greater.

2. In all probability, it not only exists where ordinary matter does not, but it also permeates all forms of matter. The motion of a medium such as water is found not to add its full value to the velocity of light moving through it, but only such a fraction of it as is perhaps accounted for on the hypothesis that the ether itself does not partake of this motion.

3. The phenomenon of the aberration of the fixed stars can be accounted for on the hypothesis that the ether does not partake of the earth's motion in its revolution about the sun. All experiments for testing this hypothesis have, however, given negative results, so that the theory may still be said to be in an unsatisfactory condition.

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PLATE I

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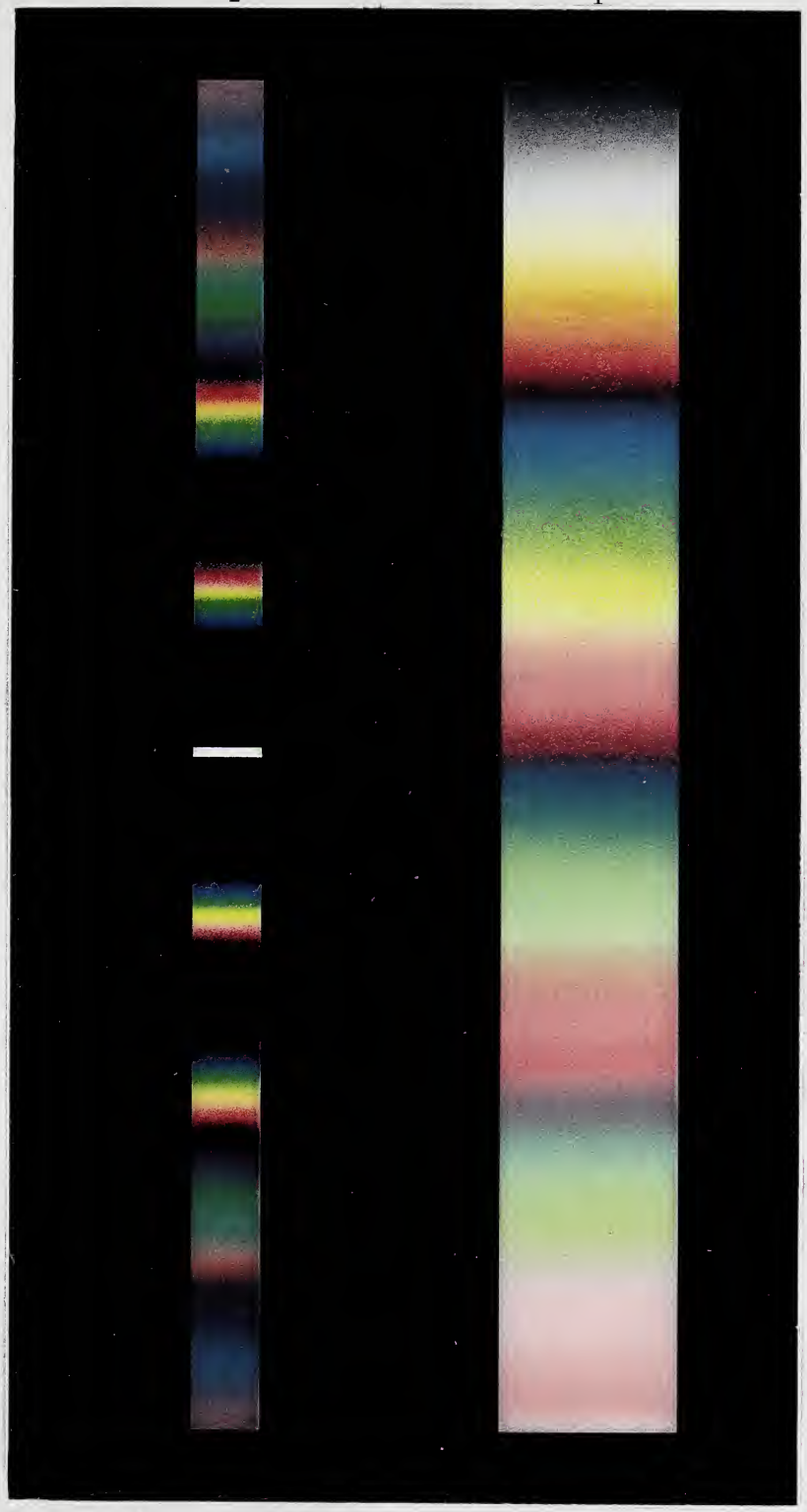

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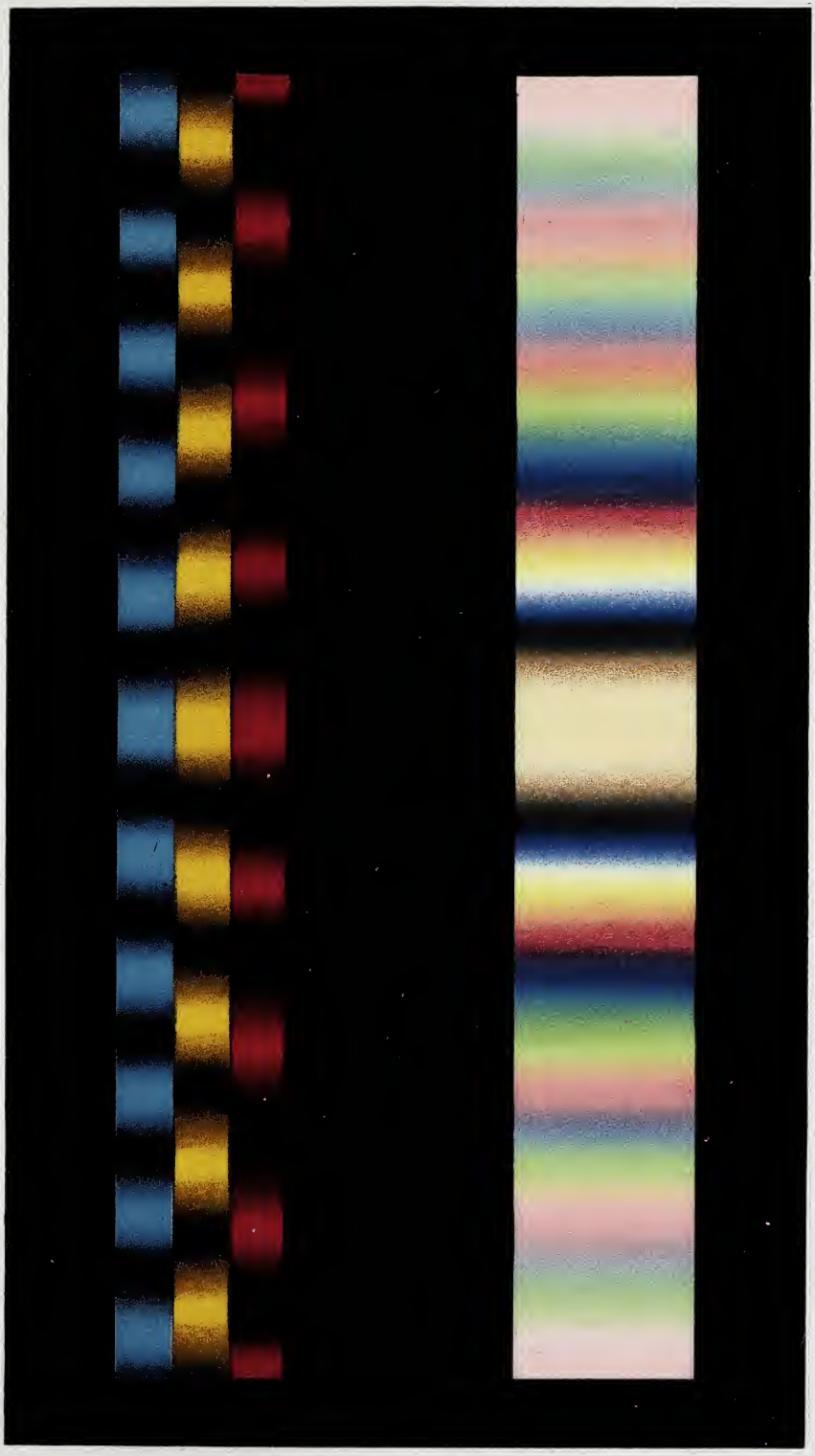

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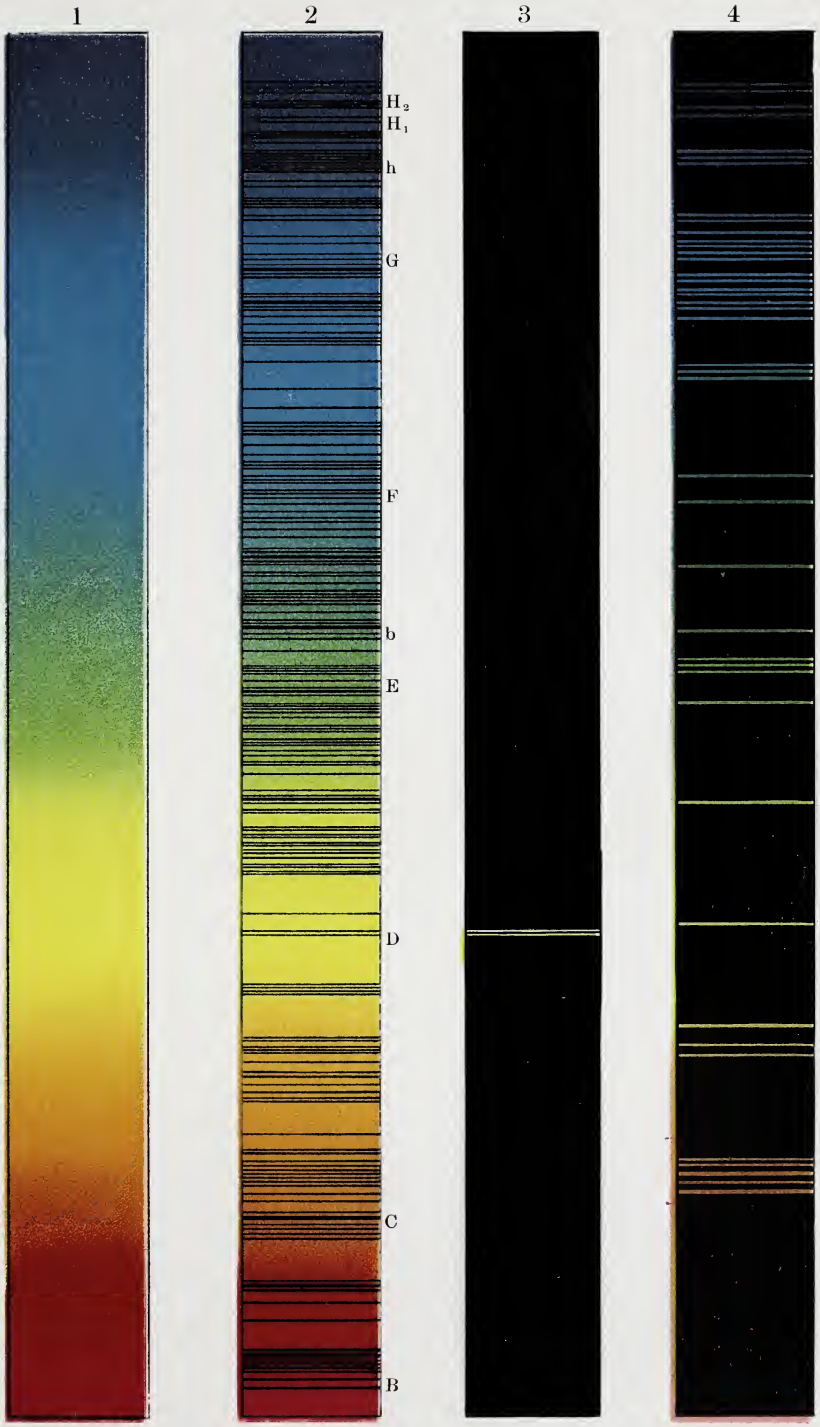

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